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BILITY OF IMPLICIT INTEGRO-DYNAMIC
EQUATION ON TIME SCALES

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EQUATION ON TIME SCALES

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EQUATION ON TIM**

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Ngày chấp nhận: $25/08/2024$ thang thời gian. Cụ thể iệt, Cầu Giấy, Hà Nội, Việt Nam

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their solutions without solving them. One of tive theory is to investigate the robust stability of systems. The robust stability is considin tính bị chặn và tính ổn định mũ.

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namic equations on time scales in mapped and problem in state, in state, in the column of stability of systems. The robust stability is considered for difference singular equations or dynamic equations on time scales in (Du, N. H. *et al* 2016; D.D. Thuan and the stress of the study is considered for difference singular equations or dynamic equations on time scales in (Du, N. H. *et al* 2016; D.D. Thuan *et al* 2019), although all most works consider only systems without o the consideration of the results of dynamic equations on time scales in (Du, N. H. et al 2016; D.D. Thuan et al 2019), although all most works consider only systems without or finite memory. Therefore, it is worth conside

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ity for the implicit integro-dynamic system on right-dens-
time scales under the form functions

$$
A(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^t K(t, s)x(s)\Delta s + f(t) \text{ from T to}
$$
with $t \geq t_0$ and $A(\cdot), B(\cdot), K(\cdot, \cdot), f(\cdot)$ are $\begin{array}{c} \text{pressure} \\ 0 \text{ (resp., 1)} \\ \text{specified later. We deal with the preservation} \end{array}$

Surface 110. No 4_August 2024| p.5
ity for the implicit integro-dynamic system on right-dense point. T
time scales under the form functions defined or
 $A(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^t K(t, s)x(s)\Delta s + f(t)$ from T to R is regre
with $t \$ ity for the implicit integro-dynamic system on right-dense point. The
time scales under the form functions defined on tl
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with $t \geq t_0$ and $A(\cdot), B(\cdot), K(\cdot, \cdot),$ time scales under the form
 $A(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^t K(t, s)x(s)\Delta s + f(t)$ from T to R is regressive (resp.

with $t \ge t_0$ and $A(\cdot), B(\cdot), K(\cdot, \cdot), f(\cdot)$ are 0 (resp., $1 + \mu(t)f(t) > 0$). We appesime if for every $t \in \mathbb{T}$, then

spe $A(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^{t} K(t, s)x(s)\Delta s + f(t)$ from T to R is
with $t \geq t_0$ and $A(\cdot), B(\cdot), K(\cdot, \cdot), f(\cdot)$ are \int_{0}^{t} (resp., $1 + \mu(t)$
specified later. We deal with the preservation $\mathcal{R}(\mathbb{T}, \mathbb{R})$ (resp.,
of the stability $A(t)x^2(t) = B(t)x(t) + \int_t^R K(t, s)x(s)\Delta s + f(t)$ from 1 to k is regressive
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of t tions. specified later. We deal with the preservation $\mathcal{R}(\mathbb{T}, \mathbb{R})$ (resp., $\mathcal{R}^+ = \mathcal{R}^+$ (resp., $\mathcal{R}(\mathbb{T}, \mathbb{R})$ (resp., $\mathcal{R}^+ = \mathcal{R}^+$)
der small perturbations. Since the derivative tions, and $C_{rd}\mathcal{R}(\math$ of the stability for this dynamic equation un-

der small perturbations. Since the derivative (resp., positively regressive)

of state process $x(t)$ at time t depends on all the set of rd-continuous (resp

past path $x(s), t$ der small perturbations. Since the derivative
of state process $x(t)$ at time t depends on all
past path $x(s), t_0 \le s \le t$, we have to use a
past path $x(s), t_0 \le s \le t$, we have to use a
more general inequality of Gronwall-Bellm

of state process $x(t)$ at time t depends on all the set of rd-continuous (
past path $x(s), t_0 \le s \le t$, we have to use a gressive) regressive functions
more general inequality of Gronwall-Bellman all $x, y \in \mathbb{T}$, we define past path $x(s), t_0 \leq s \leq t$, we have to use a different of d-conditions (response general inequality of Gronwall-Bellman and $x, y \in \mathbb{T}$, we define the *cin* type to obtain the upper bound of perturba-
tions.
The paper is more general inequality of Gronwall-Bellman
type to obtain the upper bound of perturba-
tions.
The paper is organized as follows. In the next
 $p \oplus q := p + q + \mu(t)xy$, $p \in$
The paper is organized as follows. In the next
section type to obtain the upper bound of perturba-
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The paper is organized as follows. In the next
 $p \oplus q := p + q + \mu(t)xy$,
section we recall some basic notions and pre-
it is easy to verify that,
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The paper is organized as follows. In the next

section we recall some basic notions and pre-

It is easy to verify that, for

liminary results on time scales. In section 3, $p \oplus q, p \ominus q, \ominus p, \ominus q \in \mathcal{R}$. El

we The paper is organized as follows. In the next
section we recall some basic notions and pre-
It is easy to verif
liminary results on time scales. In section 3, $p \oplus q, p \ominus q, \ominus p, \ominus q$
we consider the solvability of implic Figure is expansion and pre-
section we recall some basic notions and pre-
liminary results on time scales. In section 3, $p \oplus q, p \ominus q, \ominus p, \ominus q \in \mathcal{R}$. El
we consider the solvability of implicit integro called the inve ble. Let us a concerned with conditions such that if the solution of a implicit integro - dynamic equations is uniformly stable/exponentially stable, then under small Lipschitz perturbation it is still uniformly stable/exponent 2.1 Time scales and non-

intervalse the term of the term of the scale of the real numbers, enclosed with the term
 $\begin{array}{ccc}\n\text{2.1} & \text{Time scales} & \text{of} \\
\text{2.2} & \text{Premiliary} & \text{for} \\
\text{2.3} & \text{Time scales} & \text{of} \\
\text{A time scale is a nonempty closed subset of} & \text{the real numbers, enclosed with the topology} & \text{2$

2.1 Time scales

The section of θ . If θ is called detailed the state is still uniformly stable/exponentially sta-
 $\begin{aligned}\n\text{for all } \varepsilon > 0, \\
\|\varphi(\sigma(t)) - \varphi(s) - \varphi^{\Delta}(t)(\sigma(t))\|_{\infty}\n\end{aligned}$

2.1 Time scales

2.1 Time scales **11.13.13 Experimentally Solution** $\lVert \varphi(\sigma(t)) - \varphi(s) - \varphi^{\Delta}(t) \rVert$
 2.1 Time scales for all $s \in (t-\delta, t+\delta) \cap T$
 2.1 Time scales for the vector $\varphi^{\Delta}(t)$ is call

A time scale is a nonempty closed subset of

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 i $\varphi(\sigma(t)) - \varphi(s) - \varphi^{\Delta}(t)(\sigma(t))$
 2.1 Time scales
 2.1 Time scales
 i The vector $\varphi^{\Delta}(t)$ is called the

of f at t.

A time scale is a nonempty closed subset of

the real numbers, enclosed with the topology **2.1 Time scales** for all $s \in (t-\delta, t+\delta) \cap T$ as
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A time scale is a nonempty closed subset of

the real numbers, enclosed with the topology **2.2 Exponential F**

inherite **2.1 Time scales**
 2.1 Time scales

The vector $\varphi^{\Delta}(t)$ is called the

of f at t.

A time scale is a nonempty closed subset of

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We usually **2.1 Time scales** The vector $\varphi^{\Delta}(t)$ is called t

of f at t.

A time scale is a nonempty closed subset of

the real numbers, enclosed with the topology **2.2 Exponential F**

inherited from the standard topology on \mathbb **Example 15 of f** at t.

A time scale is a nonempty closed subset of

the real numbers, enclosed with the topology **2.2 Exponential**

inherited from the standard topology on \mathbb{R} .

We usually denote it by the symbol A time scale is a nonempty closed subset of
the real numbers, enclosed with the topology 2.2 Exponential Func
inherited from the standard topology on \mathbb{R} .
We usually denote it by the symbol \mathbb{T} . On Let \mathbb{T} the real numbers, enclosed with the topology 2.2 Exp
inherited from the standard topology on ℝ.
We usually denote it by the symbol T. On Let T be an u
the time scale T, we define the forward jump sup T = ∞.
operator σ We usually denote it by the symbol \mathbb{T} . On Let \mathbb{T} be an unbounded abo
the time scale \mathbb{T} , we define the forward jump sup $\mathbb{T} = \infty$.
operator $\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}$ and the **Definition 2.2** (Exponen
grain the time scale \mathbb{T} , we define the forward jump sup $\mathbb{T} = \infty$.
operator $\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}$ and the **Definition 2.2** (Exponential graininess $\mu(t) = \sigma(t) - t$. Similary, the back-
ward operator is defined as $\rho(t) =$ operator $\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}$ and the **Definition 2.2** (Exponenty graininess $\mu(t) = \sigma(t) - t$. Similary, the back-
ward operator is defined as $\rho(t) = \sup\{s \in \text{tion}\}\)$. Let $p : \mathbb{T} \to \mathbb{R}$ is regular operator is defined

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ity for the implicit integro-dynamic system on right-dense point. The set of
time scales under the form functions defined on the inter
 $A(t)x^{\Delta}(t) = B(t)x(t) + \int_{0}^{t} K(t, s)x(s)\Delta s + f(t$ Nguyen Thu Ha/Vol 10. No 4_August 2024|₁

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time scales under the form functions defined on the
 $A(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^t K(t, s)x(s)\Delta s + f(t)$ from T to ity for the implicit integro-dynamic system on right-dense point. The set comes time scales under the form functions defined on the integral $A(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^t K(t, s)x(s)\Delta s + f(t)$ from T to R is *regressive* (respective Ity for the implicit integro-dynamic system on right-dense point. The set of

time scales under the form functions defined on the inter
 $A(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^t K(t, s)x(s)\Delta s + f(t)$ from T to R is regressive (resp

with $t \geq$ No 4_August 2024| p.5-16
right-dense point. The set of *rd*-continuous
functions defined on the interval *J* valued in
X will be denoted by $C_{rd}(J, X)$. A function *f*
from $\mathbb T$ to $\mathbb R$ is *rearessive* (resp., *posit* No 4_August 2024| p.5-16
right-dense point. The set of *rd*-continuous
functions defined on the interval *J* valued in
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gressi right-dense point. The set of *rd*-continuous
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functions defined on the interval J valued in
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from T to R is regressive (resp., positively re-
gressive) if for every $t \in \mathbb{T}$, t Functions defined on the interval J valued in

X will be denoted by $C_{rd}(J, X)$. A function f

from T to R is *regressive* (resp., *positively re-*

gressive) if for every $t \in \mathbb{T}$, then $1+\mu(t)f(t) \neq$

0 (resp., $1+\mu(t)f(t) >$ Example denoted by $C_{rd}(J, X)$. A function f
from $\mathbb T$ to $\mathbb R$ is *regressive* (resp., *positively regressive*) if for every $t \in \mathbb T$, then $1 + \mu(t) f(t) \neq 0$ (resp., $1 + \mu(t) f(t) > 0$). We denote by $\mathcal R = \mathcal R(\mathbb T, \mathbb R)$ all x, y ∈ T, then \mathbb{R} is regressive (resp., positively regressive) if for every $t \in \mathbb{T}$, then $1 + \mu(t) f(t) \neq 0$ (resp., $1 + \mu(t) f(t) > 0$). We denote by $\mathcal{R} = \mathcal{R}(\mathbb{T}, \mathbb{R})$ (resp., $\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T}, \mathbb{R})$ gressive) if for every $t \in \mathbb{T}$, then $1+\mu(t)f(0)$ (resp., $1+\mu(t)f(t) > 0$). We denote by $\mathcal{R}(\mathbb{T}, \mathbb{R})$ (resp., $\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T}, \mathbb{R})$) the s (resp., positively regressive) regressive is tions, and $C_{rd}\mathcal{R}(\mathbb$ 0 (resp., $1 + \mu(t)f(t) > 0$). We denote by $\mathcal{R} = \mathcal{R}(\mathbb{T}, \mathbb{R})$ (resp., $\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T}, \mathbb{R})$) the set of
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 $p, q \in \mathcal{R}$,
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tions, and C_{rd} $\mathcal{R}(\mathbb{T}, \mathbb{R})$ (resp., C_{rd} $\mathcal{R}^+(\mathbb{T}, \mathbb{R})$)
the set of rd-continuous (resp., positively re-
gressive) regressive functions from T to R. For
all x

$$
p \oplus q := p + q + \mu(t)xy, \qquad p \ominus q := \frac{p - q}{1 + \mu(t)q}.
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the set of rd-continuous (resp., positively regressive) regressive functions from T to R. For
all $x, y \in \mathbb{T}$, we define the *circle plus* and he
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 $p \oplus q := p + q + \mu(t)xy$, $p \ominus q := \frac{p - q}{1 + \mu(t)q}$.
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It is easy to verify that, for all $p, q \in \mathbb{R}$,
 $p \oplus q, p \omin$ all $x, y \in \mathbb{T}$, we define the *circle plus* and he
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It is easy to verify that, for all $p, q \in \mathcal{R}$,
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It is easy to verify that, for all $p, q \in \mathcal{R}$,
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Abelian group.

(Delta Derivative). A func-

is called delta differentiable
 It is easy to verify that, for all $p, q \in \mathcal{R}$,
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ti It is easy to verity that, for all $p, q \in \mathcal{R}$,
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tion $\varphi : \mathbb{T} \to R^d$ is called delta differentiable
at t if there exists a vector $\varphi^{\Delta}(t)$ such that ftion ⊕ forms an Abelian group.
 Definition 2.1 (Delta Derivative). A func-

tion $\varphi : \mathbb{T} \to R^d$ is called delta differentiable

at t if there exists a vector $\varphi^{\Delta}(t)$ such that

for all $\varepsilon > 0$,
 $\|\varphi(\sigma(t)) - \varphi(s$

$$
|\varphi(\sigma(t)) - \varphi(s) - \varphi^{\Delta}(t)(\sigma(t) - s)| \le \varepsilon |\sigma(t) - s|
$$

of f at t. for all $\varepsilon > 0$,
 $\|\varphi(\sigma(t)) - \varphi(s) - \varphi^{\Delta}(t)(\sigma(t) - s)\| \leq \varepsilon |\sigma(t) - s|$

for all $s \in (t - \delta, t + \delta) \cap T$ and for some $\delta > 0$.

The vector $\varphi^{\Delta}(t)$ is called the delta derivative

of f at t.
 2.2 Exponential Functions

Let T $\|\varphi(\sigma(t)) - \varphi(s) - \varphi^{\Delta}(t)(\sigma(t) - s)\| \le \varepsilon |\sigma(t) - s|$
for all $s \in (t - \delta, t + \delta) \cap T$ and for some $\delta > 0$.
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of f at t.
2.2 Exponential Functions
Let T be an unbounded above ti

Solution $\psi^{\Delta}(t)$ is called the delta derivative
of f at t.
 2.2 Exponential Functions

Let \mathbb{T} be an unbounded above time scale, that is

sup $\mathbb{T} = \infty$.
 Definition 2.2 (Exponential stability fun-

tion). Let of f at t.
 2.2 Exponential Functions

Let T be an unbounded above time scale, that is

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 Definition 2.2 (Exponential stability fun-

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the exponential 2.2 Exponential Functions

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 2.2 **Exponential Functions**

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tial function: If *p. a* sup $\mathbb{T} = \infty$.
 Definition 2.2 (Exponential stability fun-

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 $e_p(t, t_0) = \exp \Big\{ \int_{t_0}^t \lim_{h \searrow \mu(s)} \frac{\text{Ln}(1 + hp(s))}{h} \Delta s \Big\}.$

Properties o **Definition 2.2** (Exponential stability fun-
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the exponential function by
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Properties of the exponential func

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e_p(t,t_0) = \exp\Big\{\int_{t_0}^t \lim_{h \searrow \mu(s)} \frac{\text{Ln}(1 + hp(s))}{h} \Delta s \Big\}.
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\n*t, r, s*
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 T then the following hold:
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e_p(t, s)e_q(t, s) = e_{p+q}(t, s).
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e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t, s);
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e_p(t, s)e_p(s, r) = e_p(t, r).
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\n**Lemma 2.5.** *A, B, e*
\n*Lemma 2.4 and suppose*
\n*nonsingular. Then, then*
\n*nonsingular. Then, then*

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 $t, r, s \in \mathbb{T}$ then the following hold:
 $e_p(t, s)e_q(t, s) = e_{p+q}(t, s).$
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 $e_p(t, s)e_p(s, r) = e_p(t, r).$
 Emma 2.5. *t*, $r, s \in \mathbb{T}$ then the following hold:
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 $e_p(t, s)e_p(s, r) = e_p(t, r).$
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 Lemma 2.4 and suppose
 Theorem 2.3 (see (Bohner, M.

$$
x^{\Delta}(t) = p(t)x(t), x(t_0) = 1.
$$

 $e_p(t, s)e_q(t, s) = e_{p+q}(t, s).$
 $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t, s);$

Lemma 2.5. A, B, Q
 $e_p(t, s)e_p(s, r) = e_p(t, r).$

Lemma 2.4 and suppose to

Theorem 2.3 (see (Bohner, M. et al 2001)). nonsingular. Then, there

If p is regressive and $t_0 \in \$ **EVALUATE:**

For any $\begin{aligned}\n &\text{Theorem 2.3 (see (Bohner, M. et al 2001))}\n &\text{nonsingular. Then, there hold} \\
 &\text{If } p \text{ is regressive and } t_0 \in \mathbb{T}, \text{ then } e_p(., t_0) \text{ is}\n &\text{a unique solution of the initial value problem} \\
 &\text{where } t \in \mathbb{T}.\n \end{aligned}\n \begin{aligned}\n &\text{In this, the result of the original value problem} \\
 &\text{where } t \in \mathbb{T}.\n \end{aligned}\n \begin{aligned}\n &\text{In this, the result of the original value$ **Theorem 2.3** (see (Bohner, M. *et al* 2001)). *nonsinguar. Then, there*
 If p is regressive and $t_0 \in \mathbb{T}$, then $e_p(., t_0)$ is
 a unique solution of the initial value problem
 $x^{\Delta}(t) = p(t)x(t), x(t_0) = 1.$

Let T be If p is regressive and $t_0 \in \mathbb{T}$, then $e_p(., t_0)$ is reduced is a unique solution of the initial value problem
 $x^{\Delta}(t) = p(t)x(t), x(t_0) = 1.$

Let \mathbb{T} be time scale that is unbounded above.

Let \mathbb{T} be time scale t a unique solution of the initial value problem
 $x^{\Delta}(t) = p(t)x(t), x(t_0) = 1.$
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Let T be time scale that is unbounded above.

Let T be time scale that is unbounded above.

For any $a, b \in \mathbb{R}$, the n $x^{\Delta}(t) = p(t)x(t), x(t_0) = 1.$ b) $-G^{-1}B T Q_{\sigma} = Q_{\sigma}.$

Let T be time scale that is unbounded above.

For any $a, b \in \mathbb{R}$, the notation $[a, b]$ or (a, b) projector, is the projectors of the segment on T, that is $[a, b] = \{t \in \mathbb{T$ $x^{\Delta}(t) = p(t)x(t), x(t_0) = 1.$ b) $-G^{-1}B TQ_{\sigma} = Q_{\sigma}$

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For any $a, b \in \mathbb{R}$, the notation $[a, b]$ or (a, b)

means the segment on T, that is $[a, b] = \{t \in \mathbb{T} : a \le t \le b\}$
 $\exists a \le t \le b\$ Let T be time scale that is unbounded above.

For any $a, b \in \mathbb{R}$, the notation $[a, b]$ or (a, b) along S.

means the segment on T, that is $[a, b] = \{t \in \mathbb{T} : a \le t \le b\}$ or $(a, b) = \{t \in \mathbb{T} : a < t < b\}$ along S.

and $\mathbb{T}_a = \$ Let T be time scale that is unbounded above.

For any $a, b \in \mathbb{R}$, the notation $[a, b]$ or (a, b) along S.

means the segment on T, that is $[a, b] = \{t \in$

T : $a \le t \le b\}$ or $(a, b) = \{t \in \mathbb{T} : a < t < b\}$ along S.

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means the segment on \mathbb{T} , that is $[a, b] = \{t \in \mathbb{T} : a \le t \le b\}$ or $(a, b) = \{t \in \mathbb{T} : a < t < b\}$
and $\mathbb{T}_a = \{t \ge a : t \in \mathbb{T}\}$. We can de-
fine a measure $\Delta_{\mathbb{T}}$ on $\$ For the notation $[a, b]$ or (a, b) along S.

the notation $[a, b]$ or (a, b) along S.

the notation $[a, b] = \{t \in \mathbb{T} : a < t < b\}$ along S.
 $a : t \in \mathbb{T}\}$. We can declare the value of T and Q.
 $\Delta_{\mathbb{T}}$ on T by considering the For any $s, b \in \mathbb{R}$, the notation [a, b]
means the segment on \mathbb{T} , that is $[a, b]$
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and $\mathbb{T}_a = \{t \ge a : t \in \mathbb{T}\}$. We c
fine a measure $\Delta_{\mathbb{T}}$ on \mathbb{T} by consideri
Caratheod and $\mu_a - \mu \geq a + t \in \pi$, we can define a measure $\Delta_{\mathbb{T}}$ on \mathbb{T} by considering the
Caratheodory construction of measures when
we put $\Delta_{\mathbb{T}}[a, b] = b - a$. The Lebesgue integral in (R. März. 1998), p.319.
of a measura We put $\Delta_{\mathbb{T}}[u, v] = v - a$. The Ecoesgiae integral
of a measurable function f with respect to $\Delta_{\mathbb{T}}$ To consider the robust stabi
is denoted by $\int_a^b f(s) \Delta_{\mathbb{T}}s$ (see (Guseinov, G. Gronwall-Bellman's inequalit
Sh. 2 Solution of a measurable function f with respect to $\Delta \mathbb{T}$. To consider the denoted by $\int_a^b f(s) \Delta_{\mathbb{T}} s$ (see (Guseinov, G. Gronwall troduced troduced **Lemma**
2.3 Some surveys on linear al-). Let the **gebra** that

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Lemma 2.6. (see (
 2.3 Some surveys on linear al-). Let the functions of
 gebra nonnegative and con

We survey briefly some basic properties of lin- $t \in \mathbb{T}_a$

ear implicit dynamic equation.

Lemma 2.4. Let A and **1.3 Some surveys on linear al-**). Let the functions $u(t)$,
 gebra $u(t)$, $u(t)$, $u(t)$ and let c_1 and c_2 be

We survey briefly some basic properties of lin- $t \in \mathbb{T}_a$

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 Lemma 2.4 2.3 Some surveys on linear al-). Let the function
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we survey briefly some basic properties of lin-
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 Lemma 2.4. Let A and B be given $n \times n$
 $u(t)$ **Example 19 Some Some Some Some Some Some Some is all to a continuous** the interpolations $u(t)$, $v(t)$ **Example is a** continuou t , and let c_1 and c_2 be near implicit dynamic equation.
 Lemma 2.4. Let A and B be **EXECUTE:**
 $\begin{aligned}\n &\text{where,}\\ \text{the survey briefly some basic properties of line} & t, \text{ and let } c_1 \text{ and } c_2 \\
 &\text{the arm implicit dynamic equation.}\n \end{aligned}\n \begin{aligned}\n &\text{L}{\mathbf{E}} \mathbf{w}^{(t)} = \mathbf{w}^{(t)} + c_1 \mathbf{w}^{(t)} + c_2 \mathbf{w}^{(t)} + c_3 \mathbf{w}^{(t)} + c_4 \mathbf{w}^{(t)} + c_5 \mathbf{w}^{(t)} + c_6 \mathbf{w}^{(t)} + c_7 \mathbf{w}^{(t)} + c$ $)$ We survey briefly some basic properties of lin-
 $t \in \mathbb{T}_a$

ear implicit dynamic equation.

Lemma 2.4. Let A and B be given $n \times n$
 $u(t) \leq \varphi(t) \left[c_1 + c_2 \int_{\tau}^t v(s) u(s) +$

matrices, and Q be a projector onto Ker A,

i.e the same of the basic properties of integral $u(t) \leq \varphi(t) \left[c_1+c_2\int_{\tau}^{t} [v(s)u(s)+$
 Lemma 2.4. Let A and B be given $n \times n$
 $u(t) \leq \varphi(t) \left[c_1+c_2\int_{\tau}^{t} [v(s)u(s)+$
 $matrixes, and Q$ be a projector onto $KerA$, then with $p(\cdot) = c_2 [v$ **Lemma 2.4.** Let A and B be given $n \times n$
matrices, and Q be a projector onto KerA,
i.e., $Q^2 = Q$, $Im Q = Ker A$. Denote $S = \{x : Bx \in Im A_{\sigma}\}$. Let T be a continuous func-
tion defined on \mathbb{T}_a , taking values in $Gl(\mathbb{R}^n)$
su b) ^G ⁼ ^A^σ [−] BT Q^σ is nonsingular.

a)
$$
S \cap \text{Ker } A = \{0\}.
$$

b)
$$
G = A_{\sigma} - BT Q_{\sigma}
$$
 is nonsingular.

c)
$$
\mathbb{R}^n = S \oplus \text{Ker }A
$$
.

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 Proof. The proof of this lemma can be found

in (R. März. 1998), Appendix 1, Lemma A1,

p.329. No 4_August 2024| p.5-16
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No 4_August 2024| p.5-16
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p.329. \Box
 Lemma 2.5. A, B, Q, G mentioned in
 Lemma 2.4 and suppose that the matrix G is

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 Lemma 2.4 and suppose that the matrix G is

nonsingular. Then, there hold the following

r relations: (i.e. Mar2. 1990), Appendix 1, Eemina A1,

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 mma 2.4 and suppose that the matrix G is

msingular. Then, there hold the following

aions:

a) $P_{\sigma} = G^{-1} A_{\sigma}$ where $P_{\sigma} = I - Q_{\sigma}$.

b) $-G^{-1} B T Q_{\sigma} = Q_{\sigma}$.

c) $\$ P_{α} and suppose that the matrix G is
ngular. Then, there hold the following
ons:
 $P_{\sigma} = G^{-1} A_{\sigma}$ where $P_{\sigma} = I - Q_{\sigma}$.
 $-G^{-1} B T Q_{\sigma} = Q_{\sigma}$.
 $\hat{Q} := -T Q_{\sigma} G^{-1} B$, called canonical
projector, is the projector onto

a)
$$
P_{\sigma} = G^{-1} A_{\sigma}
$$
 where $P_{\sigma} = I - Q_{\sigma}$.

$$
b) \qquad -G^{-1} \, B \, T Q_{\sigma} = Q_{\sigma}.
$$

- along S. ations:

a) $P_{\sigma} = G^{-1} A_{\sigma}$ where $P_{\sigma} = I - Q_{\sigma}$.

b) $-G^{-1} B T Q_{\sigma} = Q_{\sigma}$.

c) $\hat{Q} := -T Q_{\sigma} G^{-1} B$, called canonica

projector, is the projector onto Ker_A

along S.

d) $T Q_{\sigma} G^{-1}$ does not depend on the choice of A_{σ} where $P_{\sigma} = I - Q_{\sigma}$.
 $Q_{\sigma} G^{-1} B$, called canonical

the projector onto KerA

does not depend on the

und Q.

in this lemma are proved a) $P_{\sigma} = G^{-1} A_{\sigma}$ where $P_{\sigma} = I - Q_{\sigma}$.

b) $-G^{-1} B T Q_{\sigma} = Q_{\sigma}$.

c) $\hat{Q} := -T Q_{\sigma} G^{-1} B$, called canonical

projector, is the projector onto KerA

along S.

d) $T Q_{\sigma} G^{-1}$ does not depend on the

choice of T and Q.
 c) $\hat{Q} := -TQ_{\sigma} G^{-1}B$, called canonical
projector, is the projector onto KerA
along S.
d) $TQ_{\sigma} G^{-1}$ does not depend on the
choice of T and Q.
Proof. The results in this lemma are proved
in (R. März. 1998), p.319. \square
	-

projector, is the projector onto KerA
along S.
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Proof. The results in this lemma are proved
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To consider the robust stability we need along S.

d) $TQ_{\sigma}G^{-1}$ does not depend on the

choice of T and Q.

Proof. The results in this lemma are proved

in (R. März. 1998), p.319. \Box

To consider the robust stability we need the

Gronwall-Bellman's inequali d) $TQ_{\sigma}G^{-1}$ does not depend on the
choice of T and Q.
Proof. The results in this lemma are proved
in (R. März. 1998), p.319. \Box
To consider the robust stability we need the
Gronwall-Bellman's inequality. It will be

choice of T and Q.

Proof. The results in this lemma are proved

in (R. März. 1998), p.319. \Box

To consider the robust stability we need the

Gronwall-Bellman's inequality. It will be in-

troduced and applied in the fo *Proof.* The results in this lemma are proved
in (R. März. 1998), p.319. \Box
To consider the robust stability we need the
Gronwall-Bellman's inequality. It will be in-
troduced and applied in the following lemma.
Lemma *Proof.* The results in this lemma are proved
in (R. März. 1998), p.319. \Box
To consider the robust stability we need the
Gronwall-Bellman's inequality. It will be in-
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Lemma in (R. März. 1998), p.319. \Box

To consider the robust stability we need the

Gronwall-Bellman's inequality. It will be in-

troduced and applied in the following lemma.
 Lemma 2.6. (see (Choi, S. K. et al 2010)

). Le $t \in \mathbb{T}_a$ It will be in-
lowing lemma.
K. et al 2010)
 $v(t), w(t,r)$ be
 $r a \leq \tau \leq r \leq$
egative. If for
 $\int_{r}^{s} w(s,r)u(r)\Delta r \Delta s$
 $\left[\cdot, r\right)\Delta r$, **Lemma 2.6.** (see (Choi, S. K. et al 2010)

). Let the functions $u(t), \sigma(t), v(t), w(t, r)$ be

nonnegative and continuous for $a \le \tau \le r \le$

t, and let c_1 and c_2 be nonnegative. If for
 $t \in \mathbb{T}_a$
 $u(t) \le \varphi(t) \left[c_1 + c_2 \int_{\$ (*Choi*, *S. K. et al 2010*)

s $u(t), \sigma(t), v(t), w(t, r)$ be

ntinuous for $a \leq \tau \leq r \leq c_2$ be nonnegative. If for
 $\left[v(s)u(s) + \int_{\tau}^{s} w(s, r)u(r)\Delta r\right]\Delta s$
 $\left[v(\cdot) + \int_{\tau} w(\cdot, r)\Delta r\right]$,
 $)e_{p(\cdot)}(t, \tau), t \geq \tau$. $\begin{aligned} &\textit{t}(t), \textit{t}(t), \textit{t}($

$$
u(t) \leq \varphi(t) \left[c_1 + c_2 \int_{\tau}^{t} \left[v(s)u(s) + \int_{\tau}^{s} w(s,\tau)u(\tau)\Delta\tau\right]\Delta s\right]
$$

then with $p(\cdot) = c_2 \left[v(\cdot) + \int_{\tau} w(\cdot, r) \Delta r \right],$,

$$
u(t) \le c_1 \varphi(t) e_{p(\cdot)}(t,\tau), \quad t \ge \tau.
$$

$\begin{aligned} &u(t)\!\leq\!\varphi(t)\!\left[c_1\!\!+\!\!c_2\!\int_\tau^t\!\!\left[v(s)u(s)\!\!+\!\!\int_\tau^s\!\!w(s,r)u(r)\Delta r\right]\!\!\Delta s\right] \ &\text{then }\text{with }p(\cdot)=c_2\left[v(\cdot)+\int_\tau^s\!w(\cdot,r)\Delta r\right],\ &u(t)\leq c_1\varphi(t)e_{p(\cdot)}(t,\tau),\ \ t\geq\tau. \ &\text{\bf 3}\quad \text{\bf Solvability \ \ of \ implicit} \ &\text{\bf integero-dynamic \ \ equations} \end{aligned}$ $\varphi(t)\Big|c_1+c_2\int_{\tau}\Big[v(s)u(s)+\int_{\tau}w(s,r)u(r)\Delta r\Big]\Delta s\Big|$
with $p(\cdot)=c_2\Big[v(\cdot)+\int_{\tau}w(\cdot,r)\Delta r\Big],$
 $u(t)\leq c_1\varphi(t)e_{p(\cdot)}(t,\tau),\ \ t\geq \tau.$
**Solvability of implicit
integro-dynamic equa-
tions** tions $u(t) \leq c_1 \varphi(t) e_{p(\cdot)}(t,\tau), t \geq \tau.$
 **Solvability of implicit

integro-dynamic equa-

tions**

Let $A(\cdot), B(\cdot)$ be two continuous functions

tefined on \mathbb{T}_{t_0} , valued in the set of $n \times n$ -

natrices ($\mathbb{R}^{n \times n}$ for b **3 Solvability of implic integro-dynamic equality**
tions
Let $A(\cdot), B(\cdot)$ be two continuous functions
defined on \mathbb{T}_{t_0} , valued in the set of $n \times$ matrices ($\mathbb{R}^{n \times n}$ for brief), $f \in L_p^{loc}(\mathbb{T}_{t_0}; \mathbb{R}^n)$ 3 Solvability of implicit
integro-dynamic equa-
tions
Let $A(\cdot), B(\cdot)$ be two continuous functions
defined on \mathbb{T}_{t_0} , valued in the set of $n \times n$ -
matrices ($\mathbb{R}^{n \times n}$ for brief), $f \in L_p^{loc}(\mathbb{T}_{t_0}; \mathbb{R}^n)$ **ility of implicit**
 o-dynamic equa-

be two continuous functions

valued in the set of $n \times n$ -

for brief), $f \in L_p^{loc}(\mathbb{T}_{t_0}; \mathbb{R}^n)$

 $_{p}^{loc}(\mathbb{T}_{t_{0}};\mathbb{R}^{n})$

Nguyen Thu Ha/Nol 10. No 4_August 2024| p.5-16
and $K(\cdot, \cdot)$ be a two-variable continuous func- *Proof.* We divide the proot
tion defined on the set $\{(t, s) : t_0 \le s \le t < s$ teps.
 $\infty\}$, valued in $\mathbb{R}^{n \times n}$. Consider *Nguyen Thu Ha/Nol 10. No 4_August 2024* p.5-16

and $K(\cdot, \cdot)$ be a two-variable continuous func-
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tion defined on the set $\{(t, s) : t_0 \le s \le t <$ steps.
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and $K(\cdot, \cdot)$ be a two-variable continuous func-
tion defined on the set $\{(t, s) : t_0 \le s \le t < \infty\}$, valued in $\mathbb{R}^{n \times n}$. Consider the linear im-
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fo Nguyen Thu Ha/Vol 10. No 4_August 2024| p.5-16
variable continuous func-
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set $\{(t, s) : t_0 \le s \le t <$ steps.
Consider the linear im-
 \bullet Split the solution $x(\cdot)$ inter-
tions on time scales (IDE an *Nguyen Thu Ha/*Vol 10. No 4_August 2024| p.5-16

and $K(\cdot, \cdot)$ be a two-variable continuous func-
 Proof. We divide the proof
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 • Split the solution $x(\cdot)$ ir *Nguyen Thu H*
and $K(\cdot, \cdot)$ be a two-variable continuot
tion defined on the set $\{(t, s) : t_0 \le s$
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plicit dynamic equations on time scale
for short)
 $A_{\sigma}(t)x^{\Delta}(t) = B(t)x(t) + \$ Agayen Tha Ha, Not To:

And $K(\cdot, \cdot)$ be a two-variable continuous func-

tion defined on the set $\{(t, s) : t_0 \le s \le t < \infty\}$, valued in $\mathbb{R}^{n \times n}$. Consider the linear im-

plicit dynamic equations on time scales (IDE
 the continuous function of Transform $\{t, s\}$: $t_0 \le s \le t <$ steps.

independent in the solution time scales (IDE and try to solve $Qx(\cdot)$. Multiplyin $P_{\sigma}G^{-1}$, $Q_{\sigma}G^{-1}$ and $K(t, s)x(s)\Delta s + f(t)$ $P_{\sigma} = G^{-1}A_{\sigma}$, (3.1)
 and $K(\cdot, \cdot)$ be a two-variable continuous func-
 $Proof.$ We divide the proof

tion defined on the set $\{(t, s) : t_0 \le s \le t < s$ teps.
 $\infty\}$, valued in $\mathbb{R}^{n \times n}$. Consider the linear im-
 \bullet Split the solution $x(\cdot)$ in

$$
A_{\sigma}(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^{t} K(t,s)x(s)\Delta s + f(t) \qquad I_{\sigma}G \quad , \quad Q \tag{3.1}
$$

tion defined on the set $\{(t, s) : t_0 \le s \le t < s$ steps.
 $\infty\}$, valued in $\mathbb{R}^{n \times n}$. Consider the linear im-

plicit dynamic equations on time scales (IDE and try to solve $u(\cdot) = Px(\cdot)$

for short)
 $Qx(\cdot)$. Multiplying ∞ }, valued in $\mathbb{R}^{n \times n}$. Consider the linear im-
plicit dynamic equations on time scales (IDE and try to solve
for short)
 $Qx(\cdot)$. Multiplyin
 $P_{\sigma}G^{-1}$, $Q_{\sigma}G^{-1}$ at
 $A_{\sigma}(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^{t} K(t, s)x(s) \Delta$ plicit dynamic equations on time scales (IDE and try to solve $u(\cdot) =$
for short)
 $Qx(\cdot)$. Multiplying both s
 $P_{\sigma}G^{-1}$, $Q_{\sigma}G^{-1}$ and using
 $A_{\sigma}(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^t K(t, s)x(s)\Delta s + f(t)$
 $P_{\sigma} = G^{-1}A_{\sigma}$, $-G^{-}$
(3.1)
T for short)
 $Qx(\cdot)$. Multiplyin
 $P_{\sigma}G^{-1}$, $Q_{\sigma}G^{-1}$ and
 $A_{\sigma}(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^t K(t, s)x(s)\Delta s + f(t)$
 $P_{\sigma} = G^{-1}A_{\sigma}$,

(3.1)

To solve this equation, we suppose that we obtain, for $t \ge$

Ker $A(\cdot)$ is smooth in all $t \in \mathbb{T}_{t_0}$. By setting $P = I - Q$ we can rewrite the equation (3.1) as $P_{\sigma}G^{-1}, Q_{\sigma}G^{-1}$ and using th
 $= B(t)x(t) + \int_{t_0}^t K(t, s)x(s)\Delta s + f(t)$
 $P_{\sigma} = G^{-1} A_{\sigma}, \quad -G^{-1} \overline{B}$

this equation, we suppose that we obtain, for $t \ge t_0$,

smooth in the sense there exists a
 $y \Delta$ -differentiable project $A_{\sigma}(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^{R}(t, s)x(s)\Delta s + f(t)$ $P_{\sigma} = G^{-1}$

(3.1)

To solve this equation, we suppose that we obtain, for

Ker $A(\cdot)$ is smooth in the sense there exists a

continuously Δ -differentiable projector $Q(t)$ To solve this equation, we suppose there $A(\cdot)$ is smooth in the sense there exists
continuously Δ -differentiable projector Q
onto Ker $A(t)$, i.e., Q is continuously differentiable and $Q^2 = Q$, Im $Q(t) = \text{Ker } A(t)$ i nis equation, we suppose that we obtain, for

mooth in the sense there exists a
 $\alpha^t \Delta$ -differentiable projector $Q(t)$ $u^{\Delta}(t) = (P^{\Delta}t)$, i.e., Q is continuously differ-
 $Q^2 = Q$, Im $Q(t) = \text{Ker } A(t)$ for $\qquad + P_{\sigma}G^{-1$

$$
A_{\sigma}(t)(Px)^{\Delta}(t) = \overline{B}(t)x(t) + \int_{t_0}^{t} K(t,s)x(s)\Delta s
$$

$$
+ f(t), \qquad (3.2)
$$

onto Ker $A(t)$, i.e., Q is continuously differ-

entiable and $Q^2 = Q$, Im $Q(t) = \text{Ker } A(t)$ for $P_{\sigma}G^{-1} \int_{t_0}^{t} K(t, s)(u(s) + s)$

all $t \in \mathbb{T}_{t_0}$. By setting $P = I - Q$ we can $v(t) = TQ_{\sigma}G^{-1}\overline{B}u(t) + TQ_{\sigma}$

rewrite the entiable and $Q^2 = Q$, Im $Q(t) = \text{Ker } A(t)$ for

all $t \in \mathbb{T}_t$. By setting $P = I - Q$ we can $v(t) = TQ_{\sigma}G^{-1}\overline{B}u(t) + TQ_{\sigma}G$

rewrite the equation (3.1) as
 $A_{\sigma}(t)(Px)^{\Delta}(t) = \overline{B}(t)x(t) + \int_{t_0}^t K(t, s)x(s) \Delta s$
 $+ f(t),$ (3.2) Consi all $t \in \mathbb{T}_{t_0}$. By setting $P = I - Q$ we can $v(t) = TQ_{\sigma}G^{-1}\overline{B}u(t) +$
rewrite the equation (3.1) as
 $A_{\sigma}(t)(Px)^{\Delta}(t) = \overline{B}(t)x(t) + \int_{t_0}^t K(t,s)x(s)ds + TQ_{\sigma}G^{-1}\int_{t_0}^t K(t,s)(u + f(t)),$
(3.2) • Consider the equation
where $\overline{B} := B +$ that the equation (3.1) as
 $A_{\sigma}(t)(Px)^{\Delta}(t) = \overline{B}(t)x(t) + \int_{t_0}^t K(t,s)x(s)\Delta s$
 $+ f(t),$ (3.2) • Consider to

where $\overline{B} := B + A_{\sigma}P^{\Delta}$. It is seen that the so-given by

lution $x(\cdot)$ of the equation (3.2), if it exists, is

n $A_{\sigma}(t)(Px)^{\Delta}(t) = \overline{B}(t)x(t) + \int_{t_0}^{t} K(t, s)x(s)\Delta s$
+ $f(t)$, (3.2) • Cons
operate
where $\overline{B} := B + A_{\sigma}P^{\Delta}$. It is seen that the sogiven b
lution $x(\cdot)$ of the equation (3.2), if it exists, is
not necessarily differentiable $A_{\sigma}(t)(Px)$ $(t) = B(t)x(t) + \int_{t_0}^{R} K(t,s)x(s) \Delta s$
 $+ f(t),$ (3.2)

where $\overline{B} := B + A_{\sigma}P^{\Delta}$. It is seen that the so-

lution $x(\cdot)$ of the equation (3.2), if it exists, is

not necessarily differentiable but it is required H
 + $f(t)$, (3.2) • Consider the equation (
operator $H : C([t_0, \infty); \mathbb{R}^3)$
equation (3.2), if it exists, is
ferentiable but it is required $(Hv)(t) = v(t) - TQ_{\sigma}G^{-1}\int_{t_0}^t F(t)dt$
or $Tx(\cdot)$ is Δ -differentiable
e on \mathbb{T}_{t_0 where $\overline{B} := B + A_{\sigma} P^{\Delta}$. It is seen that the so-
poerator $H : C$
lution $x(\cdot)$ of the equation (3.2), if it exists, is
not necessarily differentiable but it is required $(Hv)(t) = v(t) - t$
that the component $Px(\cdot)$ is Δ -di where $B := B + A_{\sigma} P^{\Delta}$. It is seen that the so-given by

lution $x(\cdot)$ of the equation (3.2), if it exists, is

not necessarily differentiable but it is required $(Hv)(t) = v(t) - TQ_{\sigma}G^{-1}\int_{t_0}^t K(t,$

that the component $Px(\cdot)$

 $\frac{1}{A}(\mathbb{T}_{t_0}; \mathbb{R}^n)$ is the set of tinui $y \in C(\mathbb{T}_{t_0}; \mathbb{R}^n)$ such that $P(y(\cdot))$ is chat the component $T(x)$ is Δ -differentiable
almost everywhere on \mathbb{T}_t .
Consider the space $C_A^1(\mathbb{T}_{t_0}; \mathbb{R}^n)$ is the set of tinuity of $TQ_{\sigma}G^{-1}$
everywhere- differentiable on \mathbb{T}_t .
Define the linear op Consider the space $C_A^1(\mathbb{T}_{t_0}; \mathbb{R}^n)$ is the set of trivially of $TQ_{\sigma}G$
 $y \in C(\mathbb{T}_{t_0}; \mathbb{R}^n)$ such that $Py(\cdot)$ is almost vertibility of H be

everywhere- differentiable on \mathbb{T}_t ,

Define the linear opera

 $loc(\mathbb{T}^n$. $\mathbb{R}^{n \times n}$ $\int_{\infty}^{loc} (\mathbb{T}_{t_0}; \mathbb{R}^{n \times n}).$

$$
y \in C(\mathbb{I}_{t_0}; \mathbb{K}^n) \text{ such that } Py(\cdot) \text{ is almost } \text{vertibility of } H \text{ because } (\text{everywhere- differentiable on } \mathbb{T}_t, \text{ is a Volterra integral } \text{equ})
$$
\nDefine the linear operators $G := A_{\sigma} - \overline{B}TQ_{\sigma}$. Precisely, \nIt is clear that $G \in L_{\infty}^{loc}(\mathbb{T}_{t_0}; \mathbb{R}^{n \times n})$.\n\n**Definition 3.1.** The *IDE* (3.1) is said to be $(H^{-1}y)(t) = y(t) + \sum_{n=1}^{\infty} \int_{t_0}^{t} \text{index-1 if } G(t) \text{ is invertible for all } t \in \mathbb{T}_t$.\n\nFor any $T > t_0$, consider two subspaces: where, K_n is defined by $C_{TQ_{\sigma}G^{-1}([t_0, T); \mathbb{R}^n)} = \{v \in C([t_0, T); \mathbb{R}^n) : K_1(t, s) = TQ_{\sigma}G^{-1}(t)K \quad v(t) \in \text{Im } TQ_{\sigma}G^{-1}(t)\}, \quad K_{n+1}(t, s) = \int_{s}^{t} K_n(t, \tau) \int_{t}^{t} K_n(t, \tau) \, dt \leq \text{Im } P(t) \}.$ \n\n**Theorem 3.2.** For any $t_0 \geq 0$ and $x_0 \in \text{ for any } T > t_0$ the follow \mathbb{R}^n , the equation (3.2) has a unique solution $\text{sup. } \text{sup. } \text{sup. } \mathbb{E}_T |\left[K_n(t, s)\right| \leq \text{dim } \mathbb{E}_T$ and \mathbb{E}_T is a unique solution $\text{sup. } \text{sup. } \text{$

 \mathbb{R}^n , the equation (3.2) h $C_P([t_0, T); \mathbb{R}^n) = \{u \in C([t_0, T); \mathbb{R}^n) : u(t) \in \text{Im } P(t)\}.$

Theorem 3.2. For any $t_0 \ge 0$ and $x \mathbb{R}^n$, the equation (3.2) has a unique solu $x(\cdot) \in C_A^1(\mathbb{T}_{t_0}; \mathbb{R}^n)$, satisfying the initial dition
 $P(t_0)(x(t_0) - x$ $\frac{1}{A}(\mathbb{T}_{t_0}; \mathbb{R}^n)$, satisfying the initi dition $v(t) \in \text{Im } TQ_{\sigma}G^{-1}(t)$,
 $v(t) \in \text{Im } TQ_{\sigma}G^{-1}(t)$,
 $E^{n}) = \{u \in C([t_{0}, T); \mathbb{R}^{n}) : \newline u(t) \in \text{Im } P(t)\}.$
 $v(t) \in \text{Im } P(t) \}.$
 $v(t) \$

$$
P(t_0)(x(t_0) - x_0) = 0. \t(3.3)
$$

Vo 4_August 2024| p.5-16
 Proof. We divide the proof of Theorem into

steps.

• Split the solution $x(\cdot)$ into $Px(\cdot) + Qx(\cdot)$

and try to solve $u(\cdot) = Px(\cdot)$ and $v(\cdot) =$ steps.

Vo 4_August 2024| p.5-16

Proof. We divide the proof of Theorem into

steps.

• Split the solution $x(\cdot)$ into $Px(\cdot) + Qx(\cdot)$

and try to solve $u(\cdot) = Px(\cdot)$ and $v(\cdot) = Qx(\cdot)$. Multiplying both sides of (3.2) with
 $P_{\sigma}G^{-1}$ No 4_August 2024| p.5-16
 Proof. We divide the proof of Theorem into

steps.

• Split the solution $x(\cdot)$ into $Px(\cdot) + Qx(\cdot)$

and try to solve $u(\cdot) = Px(\cdot)$ and $v(\cdot) = Qx(\cdot)$. Multiplying both sides of (3.2) with
 $P_{\sigma}G^{-1$ No 4_August 2024| p.5-16
 Proof. We divide the proof of Theorem into

steps.

• Split the solution $x(\cdot)$ into $Px(\cdot) + Qx(\cdot)$

and try to solve $u(\cdot) = Px(\cdot)$ and $v(\cdot) = Qx(\cdot)$. Multiplying both sides of (3.2) with
 $P_{\sigma}G^{-1$ $P_{\sigma}G^{-1}$, $Q_{\sigma}G^{-1}$ and using the properties quast 2024| p.5-16

We divide the proof of Theorem in
 \therefore the solution $x(\cdot)$ into $Px(\cdot) + Qx$

y to solve $u(\cdot) = Px(\cdot)$ and $v(\cdot)$

Multiplying both sides of (3.2) wi
 $Q_{\sigma}G^{-1}$ and using the properties
 $= G^{-1}A_{\sigma}$, p.5-16

e the proof of Theorem into

tion $x(\cdot)$ into $Px(\cdot) + Qx(\cdot)$

e $u(\cdot) = Px(\cdot)$ and $v(\cdot) =$

ing both sides of (3.2) with

and using the properties
 \cdot , $-G^{-1}\overline{B} TQ_{\sigma} = Q_{\sigma}$, Proof. We divide the proof of Theorem into
teps.

Split the solution $x(\cdot)$ into $Px(\cdot) + Qx(\cdot)$

and try to solve $u(\cdot) = Px(\cdot)$ and $v(\cdot) =$
 $2x(\cdot)$. Multiplying both sides of (3.2) with
 $P_{\sigma}G^{-1}$, $Q_{\sigma}G^{-1}$ and using th

$$
P_{\sigma} = G^{-1} A_{\sigma}, \qquad -G^{-1} \overline{B} T Q_{\sigma} = Q_{\sigma},
$$

the linear im- • Split the solution
$$
x(\cdot)
$$
 into $Px(\cdot) + Qx(\cdot)$
\nme scales (IDE and try to solve $u(\cdot) = Px(\cdot)$ and $v(\cdot) = Qx(\cdot)$. Multiplying both sides of (3.2) with
\n P_oG^{-1}, Q_oG^{-1} and using the properties
\ns) $x(s)\Delta s+f(t)$
\n $P_{\sigma} = G^{-1}A_{\sigma}, -G^{-1}\overline{B}TQ_{\sigma} = Q_{\sigma},$
\n(3.1)
\nsuppose that we obtain, for $t \ge t_0$,
\ne there exists a
\nprojector $Q(t)$ $u^{\Delta}(t) = (P^{\Delta} + P_{\sigma}G^{-1}\overline{B})(t)u(t) + P_{\sigma}G^{-1}f(t)$
\nmuuously differ-
\n $= \text{Ker } A(t)$ for $+P_{\sigma}G^{-1}\int_{t_0}^t K(t, s)(u(s) + v(s))\Delta s$, (3.4)
\n $I-Q$ we can $v(t) = TQ_{\sigma}G^{-1}\overline{B}u(t) + TQ_{\sigma}G^{-1}f(t)$
\n $+TQ_{\sigma}G^{-1}\int_{t_0}^t K(t, s)(u(s) + v(s))\Delta s$, (3.5)
\n $\int_{t_0}^t K(t, s)x(s)\Delta s$
\n(3.2) • Consider the equation (3.5) by defining an
\noperator $H : C([t_0, \infty); \mathbb{R}^n) \rightarrow C([t_0, \infty); \mathbb{R}^n)$
\nen that the so- given by
\n), if it exists, is
\nut it is required
\n $(Hv)(t)=v(t)-TQ_{\sigma}G^{-1}\int_{t_0}^t K(t, s)v(s)\Delta s$. (3.6)
\n $-\text{differentiable}$
\nFrom [?, Theorem 3.1] it follows that the con-
\n•) is the set of t in

• Consider the equation (3.5) by defining an operator $H: C([t_0,\infty);\mathbb{R}^n) \to C([t_0,\infty);\mathbb{R}^n)$

$$
dv(t) = v(t) - TQ_{\sigma}G^{-1} \int_{t_0}^{t} K(t, s)v(s) \Delta s. \tag{3.6}
$$

 $i(t, s)x(s)\Delta s$

(3.2) • Consider the equation (3.5)

operator $H : C([t_0, \infty); \mathbb{R}^n) \rightarrow$

that the sogiven by

if it exists, is

it is required $(Hv)(t)=v(t)-TQ_{\sigma}G^{-1}\int_{t_0}^t K(t, s)$

lifferentiable

From [?, Theorem 3.1] it follow not necessarily dimerentiable but it is required $(Hv)(t) = v(t) - TQ_{\sigma}G^{-1} \int_{t_0} K(t, s)$

almost everywhere on \mathbb{T}_t .

Consider the space $C_A^1(\mathbb{T}_t, \mathbb{R}^n)$ is the set of tinuity of $TQ_{\sigma}G^{-1}(\cdot)K(\cdot, \cdot)$
 $y \in C(\mathbb{T}_t, \math$ • Theorem $\begin{aligned} &\int_{t_0}^{R} (t, s) (\alpha(s) + v(s)) \Delta s, \quad (3.9) \\ &\text{\textcolor{red}{\bullet}} \text{Consider the equation } (3.5) \text{ by defining an operator } H: C([t_0, \infty); \mathbb{R}^n) \to C([t_0, \infty); \mathbb{R}^n) \\ &\text{given by} \\ &\text{for } H(v) = (t) - TQ_{\sigma}G^{-1} \int_{t_0}^t K(t, s)v(s) \Delta s. \quad (3.6) \\ &\text{From } [? \text{ Theorem 3.1}] \text{ it follows that the continuity of } TQ_{\sigma}$ • Consider the equation (3.5) by defining an
operator $H : C([t_0, \infty); \mathbb{R}^n) \to C([t_0, \infty); \mathbb{R}^n)$
given by
 $Iv)(t)=v(t)-TQ_{\sigma}G^{-1}\int_{t_0}^t K(t, s)v(s)\Delta s$. (3.6)
From [?, Theorem 3.1] it follows that the con-
tinuity of $TQ_{\sigma}G$ (3.5) by defining an
 ∞); \mathbb{R}^n) $\to C([t_0, \infty); \mathbb{R}^n)$
 $\int_{t_0}^{t-1} \int_{t_0}^{t} K(t, s) v(s) \Delta s$. (3.6)

.1] it follows that the con-
 $(\cdot)K(\cdot, \cdot)$ implies the in-

use $(Hv)(t) = y(t), t \ge t_0$

l equation of second kind. • Consider the equation (3.5) by defining an
operator $H : C([t_0, \infty); \mathbb{R}^n) \to C([t_0, \infty); \mathbb{R}^n)$
given by
 $\{dv\}(t) = v(t) - TQ_{\sigma}G^{-1}\int_{t_0}^t K(t, s)v(s)\Delta s.$ (3.6)
From [?, Theorem 3.1] it follows that the con-
tinuity of TQ_{σ operator $H: C([t_0, \infty); \mathbb{R}^n) \to C([t_0, \infty); \mathbb{R}^n)$
given by
 $Iv)(t)=v(t)-TQ_{\sigma}G^{-1}\int_{t_0}^t K(t, s)v(s)\Delta s.$ (3.6)
From [?, Theorem 3.1] it follows that the con-
tinuity of $TQ_{\sigma}G^{-1}(\cdot)K(\cdot, \cdot)$ implies the in-
vertibility of Precisely, t)= $v(t) - TQ_{\sigma}G^{-1} \int_{t_0}^{t_0} K(t, s)v(s) \Delta s$. (3.6)

m [?, Theorem 3.1] it follows that the con-

ity of $TQ_{\sigma}G^{-1}(\cdot)K(\cdot, \cdot)$ implies the in-

ibility of H because $(Hv)(t) = y(t), t \ge t_0$

Volterra integral equation of second ki $K(t, s)v(s)\Delta s$. (3.6)

t_{to}

t follows that the con-
 $K(\cdot, \cdot)$ implies the in-
 $(Hv)(t) = y(t), t \ge t_0$

uation of second kind.
 $\int_{t_0}^t K_n(t, s)y(s)\Delta s$ (3.7)

t_{to}

induction From [*t*, 1 neorem 3.1] it follows that the con-
tinuity of $TQ_{\sigma}G^{-1}(\cdot)K(\cdot, \cdot)$ implies the in-
vertibility of H because $(Hv)(t) = y(t), t \ge t_0$
is a Volterra integral equation of second kind.
Precisely,
 $H^{-1}y)(t) = y(t) + \sum_{n=$

$$
e(H^{-1}y)(t) = y(t) + \sum_{n=1}^{\infty} \int_{t_0}^{t} K_n(t,s)y(s)\Delta s \quad (3.7)
$$

almost vertibility of *H* because
$$
(Hv)(t) = y(t), t \ge t_0
$$

\nis a Volterra integral equation of second kind.
\n $\overline{B}TQ_{\sigma}$. Precisely,
\n $mid\ to\ be\ (H^{-1}y)(t) = y(t) + \sum_{n=1}^{\infty} \int_{t_0}^t K_n(t,s)y(s) \Delta s$ (3.7)
\n $\in \mathbb{T}_{t_0}$.
\nwees: where, K_n is defined by induction
\n \mathbb{R}^n): $K_1(t,s) = TQ_{\sigma}G^{-1}(t)K(t,s),$
\n $\begin{cases}\n1(t)\\n\end{cases}$, $K_{n+1}(t,s) = \int_s^t K_n(t,\tau)TQ_{\sigma}G^{-1}(\tau)K(\tau,s)\Delta \tau,$
\nfor $t \ge s \ge t_0$, $n \ge 1$. Paying attention that
\n $ud\ x_0 \in \text{ for any } T > t_0$ the following inequality holds
\nsolution
\n $sup_{t_0 \le s \le t \le T} ||K_n(t,s)|| \le$
\n(3.3) $\begin{cases}\n\sup_{t_0 \le s \le t \le T} ||TQ_{\sigma}G^{-1}(t)K(t,s)|| \end{cases}$ $\int_{n}^n \frac{(T-t_0)^n}{n!}$,

$$
u = \{v \in C([t_0, T); \mathbb{R}^n) : \quad K_1(t, s) = TQ_σG^{-1}(t)K(t, s),
$$

\n
$$
v(t) \in \text{Im } TQ_σG^{-1}(t)\}, \quad K_{n+1}(t, s) = \int_s^t K_n(t, \tau)TQ_σG^{-1}(\tau)K(\tau, s)\Delta\tau,
$$

\n
$$
u(t) \in \text{Im } P(t)\}.
$$

\n
$$
u(t) \in \text{Im } P(t)\}.
$$

\nfor $t \ge s \ge t_0, n \ge 1$. Paying attention that
\nfor any $t_0 \ge 0$ and $x_0 \in \text{for any } T > t_0$ the following inequality holds
\n(3.2) has a unique solution
\n
$$
satisfying the initial con-
$$

\n
$$
u(t) = \sum_{t_0 \le s \le t \le T} ||K_n(t, s)|| \le
$$

\n
$$
u(t) = \sum_{t_0 \le s \le t \le T} ||K_n(t, s)|| \le
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\n
$$
u(t) = \sum_{t_0 \le s \le t \le T} ||K_n(t, s)|| \le
$$

\n
$$
u(t) = \sum_{t_0 \le s \le t \le T} ||K_n(t, s)|| \le
$$

\n
$$
u(t) = \sum_{t_0 \le s \le t \le T} ||K_n(t, s)|| \le
$$

which implies that the serie s ^R(t, s) = ^I ⁺[∞] n=1

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which implies that the serie s
 $[t_0, t]$ for every $u \in C_P([t_0, T])$
 isfies the Lipschitz condition
 $R(t, s) = I + \sum_{n=1}^{\infty} K_n(t, s)$.
 $\text{constant } k > 0 \text{ such that}$
 $||S(t, y_1) - S(t, y_2)|| \le k$ such that the series $[t_0, t]$ for every $u \in C_P([t_0, T_0, t_0])$
 $R(t, s) = I + \sum_{n=1}^{\infty} K_n(t, s).$
 $\text{is uniformly convergent on the set } \{(t, s)\}$

such that $t_0 \leq s \leq t \leq T$ and $R(\cdot, \cdot)$ is con-for all $t \in [t_0, T]$, $y_1, y_2 \in \text{tinuous. Thus, } H^{-1}$ is also a s which implies that the serie s
 $R(t, s) = I + \sum_{n=1}^{\infty} K_n(t, s).$

is uniformly convergent on the set $\{(t, s) \in \mathbb{R}^d : t_0 \le s \le t \le T \text{ and } R(\cdot, \cdot) \text{ is continuous. Thus, } H^{-1} \text{ is also a second kind, it is is a common to } R(\cdot, \cdot)$.

This means that H is a continuous bijectio he serie s
 $[t_0, t]$ for every $u \in C_P([t_0, T])$

is fies the Lipschitz condition
 \therefore
 $\sum_{n=1}^{\infty} K_n(t, s).$
 $\|S(t, y_1) - S(t, y_2)\| \le k \sup_{t_0 \le s \le t_1}$

gent on the set $\{(t, s)\}$
 $t \le T$ and $R(\cdot, \cdot)$ is con-

for all $t \in [t_$ which implies that the serie s
 $[t_0, t]$ for every $u \in C_P$
 $R(t, s) = I + \sum_{n=1}^{\infty} K_n(t, s)$.
 $\qquad \qquad \text{constant } k > 0 \text{ such that}$
 $||S(t, y_1) - S(t, y_2)|| \leq k$

such that $t_0 \leq s \leq t \leq T$ and $R(\cdot, \cdot)$ is con-

for all $t \in [t_0, T], y_1$,

tinu $R(t, s) = I + \sum_{n=1}^{\infty} K_n(t, s).$
is uniformly convergent on the set $\{(t, s)$
such that $t_0 \le s \le t \le T$ and $R(\cdot, \cdot)$ is con
tinuous. Thus, H^{-1} is also a second kind lin
ear Volterra operator with the kernel $R(\cdot, \cdot)$
This m on $C([t_0,T];\mathbb{R}^n)$. $K(t,s) = I + \sum_{n=1} K_n(t,s).$

is uniformly convergent on the set $\{(t,s)\}$

such that $t_0 \le s \le t \le T$ and $R(\cdot, \cdot)$ is con- for all $t \in [t_0, T]$, y_1, y_2

tinuous. Thus, H^{-1} is also a second kind lin- Then the equation

ear V is uniformly convergent on the set $\{(t, s)\}$
such that $t_0 \le s \le t \le T$ and $R(\cdot, \cdot)$ is con-
for all $t \in$
tinuous. Thus, H^{-1} is also a second kind lin-
finition we perform with the kernel $R(\cdot, \cdot)$.
This means that H such that $t_0 \le s \le t \le T$ and $R(\cdot, \cdot)$ is continuous. Thus, H^{-1} is also a second kind lin-
ear Volterra operator with the kernel $R(\cdot, \cdot)$.
This means that H is a continuous bijection
on $C([t_0, T]; \mathbb{R}^n)$.
• We now t and $R(\cdot, \cdot)$ is con-
for all $t \in [t_0, T]$,
the second kind lin-
the kernel $R(\cdot, \cdot)$.
tinuous bijection $y^{\Delta} = (P^{\Delta} + P_{\sigma}G^{-1}\overline{B})y$
the form of (3.5). with the initial cond
a unique solution if
over, there exists a c

ear Volterra operator with the kernel
$$
R(\cdot, \cdot)
$$
.
\nThis means that *H* is a continuous bijection $y^{\Delta} = (P^{\Delta} + P_{\sigma}G^{-}$
\non $C([t_0, T]; \mathbb{R}^n)$.
\n• We now try to simplify the form of (3.5).
\nFrom this equation we get
\n $v(t) = H^{-1}TQ_{\sigma}G^{-1} \Big[\overline{B}u + \int_{t_0}^{t} K(\cdot, s)u(s)ds\Big](t)$
\n $+ (H^{-1}TQ_{\sigma}G^{-1}f)(t).$
\nIn noting that
\n $H^{-1}TQ_{\sigma}G^{-1} \Big(\int_{t_0}^{t} K(\cdot, s)u(s)ds\Big)(t)$
\n $H^{-1}TQ_{\sigma}G^{-1} \Big(\int_{t_0}^{t} K(\cdot, s)u(s)ds\Big)(t)$
\n $= H^{-1}(u - Hu)(t) = (H^{-1}u)(t) - u(t),$
\nwe can rewrite (3.8) as
\n $= (H^{-1}\hat{P}u)(t)$
\n $v(t) = (H^{-1}\hat{P}u)(t) - u(t) + (H^{-1}Q_{\sigma}G^{-1}f)(t)$
\nfor $t \ge t_0$. The

From this equation we get
\n
$$
v(t) = H^{-1}TQ_{\sigma}G^{-1} \Big[\overline{B}u + \int_{t_0}^{t} K(\cdot, s)u(s)ds \Big](t) \qquad and \quad z(t) \text{ are two s}
$$
\n
$$
+ (H^{-1}TQ_{\sigma}G^{-1}f)(t). \qquad (3.8)
$$
\nIn noting that
\n
$$
H^{-1}TQ_{\sigma}G^{-1} \Big(\int_{t_0}^{t} K(\cdot, s)u(s)ds \Big)(t) \qquad (3.10) \text{ has a unique condition } u(t_0) =
$$
\n
$$
= H^{-1}(u - Hu)(t) = (H^{-1}u)(t) - u(t),
$$
\nwe can rewrite (3.8) as
\n
$$
v(t) = (H^{-1}\hat{P}u)(t) - u(t) + (H^{-1}Q_{\sigma}G^{-1}f)(t) \qquad \text{for } t \ge t_0. \text{ The pre}
$$
\n
$$
u(t_0) = \begin{cases} 3.9 \end{cases}
$$
\nwhere $\hat{Q}(t) = I - \hat{P}(t) = -TQ_{\sigma}G^{-1}\overline{B}(t)$ is *pling procedure, we* the canonical projector onto Ker $A(t)$.
\n
$$
u(t_0) = P(t_0)x_0, \text{ so}
$$
\n
$$
= (H^{-1}\hat{P}u)(t) - u(t_0) + (H^{-1}Q_{\sigma}G^{-1}f)(t_0) \qquad \text{for } t \ge t_0. \text{ The procedure, we}
$$
\n
$$
= \begin{cases} 3.9 \end{cases}
$$
\n
$$
u(t_0) = P(t_0)x_0, \text{ so}
$$
\n
$$
= \begin{cases} 3.9 \end{cases}
$$
\n
$$
u(t_0) = P(t_0)x_0, \text{ so}
$$
\n
$$
= \begin{cases} 3.9 \end{cases}
$$
\n
$$
u(t_0) = P(t_0)x_0, \text{ so}
$$

$$
= H^{-1}(u - Hu)(t) = (H^{-1}u)(t) - u(t),
$$
\nwe can rewrite (3.8) as
\n
$$
v(t) = (H^{-1}\hat{P}u)(t) - u(t) + (H^{-1}Q_{\sigma}G^{-1}f)(t)
$$
\n
$$
v(t) = (H^{-1}\hat{P}u)(t) - u(t) + (H^{-1}Q_{\sigma}G^{-1}f)(t)
$$
\n
$$
v(t) = (H^{-1}\hat{P}u)(t) - u(t) + (H^{-1}Q_{\sigma}G^{-1}f)(t)
$$
\n
$$
v(t) = (H^{-1}\hat{P}u)(t) - u(t) + (H^{-1}Q_{\sigma}G^{-1}f)(t)
$$
\n
$$
v(t) = (H^{-1}\hat{P}u)(t)
$$
\n
$$
v(t) = (H^{-1}\hat{P}u)(t) + H^{-1}\hat{P}u(t)
$$
\n
$$
v(t) = (H^{-1}\hat{P}u)(t) - u(t) + (H^{-1}\hat{Q}u)(t)
$$
\n
$$
u(t) = H^{-1}\hat{P}u(t) + u(t) + u(t)
$$
\n

we can rewrite (3.6) as
\n
$$
= (H^{-1}\hat{P}u)(t) + (H^{-1}Q_{\sigma}G^{-1}f)(t)
$$
\nfor $t \geq t_0$. The proof is comp
\n(3.9)
\nwhere $\hat{Q}(t) = I - \hat{P}(t) = -TQ_{\sigma}G^{-1}\overline{B}(t)$ is *pling procedure, we state the*
\nthe canonical projector onto Ker $A(t)$.
\nSubstituting $v(t)$ into (3.4) obtains
\n $u^{\Delta}(t) = (P^{\Delta} + P_{\sigma}G^{-1}\overline{B})u(t) + P_{\sigma}G^{-1}f(t)$
\n $+ P_{\sigma}G^{-1}\int_{t_0}^t K(t, s)H^{-1}(\hat{P}u + TQ_{\sigma}G^{-1}f)(s)\Delta s$ *pend on the chosen projector*
\nfor all $t \geq t_0$.
\nWe note that the condition (3
\n $+ P_{\sigma}G^{-1}\int_{t_0}^t K(t, s)H^{-1}(\hat{P}u + TQ_{\sigma}G^{-1}f)(s)\Delta s$ *pend on the chosen projector*
\nfor all $t \geq t_0$.
\n(3.10) (3.10). Multiplying both sides
\nwith Q_{σ} yields $Q_{\sigma}u^{\Delta} = Q_{\sigma}I$
\nWe now use the following lemma, its proof *tention that* $Q^{\Delta} = (Q^2)^{\Delta} =$
\ncan be easily obtained by using Picard's ap-*obtains*
\nproximation method and usual procedures.
\n**Lemma 3.3.** Let *S* be a function defined on *Thus, if* $Q(t_0)u(t_0) = 0$ the
\n $[t_0, T] \times C_P([t_0, T]; \mathbb{R}^n)$, valued in \mathbb{R}^n , such for all $t \geq t_0$. This means that
\nthat $S(t, u)$ depends only the values of u on invariant property: every solu

*S*_{to} *ii*) Let $u(t)$ be the solu
for all $t \ge t_0$. (3.10) (3.10). Multiplying both *i with* Q_{σ} yields $Q_{\sigma}u^{\Delta} =$
We now use the following lemma, its proof *tention that* $Q^{\Delta} = (Q^2)$
can be easily obtained b

 $([t_0, T]: \mathbb{R}^n)$, valued in \mathbb{R}^n , such for all $t > t_0$

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e s $[t_0, t]$ for every $u \in C$

isfies the Lipschitz
 $K_n(t, s)$.
 $||S(t, y_1) - S(t, y_2)||$

n the set $\{(t, s)\}$ No 4_August 2024| p.5-16
[t₀, t] for every $u \in C_P([t_0, T]; \mathbb{R}^n)$ and *S* satisfies the Lipschitz condition, i.e., there is constant $k > 0$ such that $([t_0, T]; \mathbb{R}^n)$ and S sat- $[t_0, T]; \mathbb{R}^n$ and S sat-
dition, i.e., there is a
it No 4_August 2024| p.5-16

[t₀, t] for every $u \in C_P([t_0, T]; \mathbb{R}^n)$ and S satisfies the Lipschitz condition, i.e., there is a

constant $k > 0$ such that
 $||S(t, y_1) - S(t, y_2)|| \le k$ sup $||y_1(s) - y_2(s)||$, No 4_August 2024| p.5-16

[t₀, t] for every $u \in C_P([t_0, T]; \mathbb{R}^n)$ and S sat-

isfies the Lipschitz condition, i.e., there is a

constant $k > 0$ such that
 $||S(t, y_1) - S(t, y_2)|| \le k \sup_{t_0 \le s \le t} ||y_1(s) - y_2(s)||$, No 4_August 2024| p.5-16

[t₀, t] for every $u \in C_P([t_0, T]; \mathbb{R}^n)$ and S sat-

isfies the Lipschitz condition, i.e., there is a

constant $k > 0$ such that
 $||S(t, y_1) - S(t, y_2)|| \le k \sup_{t_0 \le s \le t} ||y_1(s) - y_2(s)||$,

for all $t \in [$ $[t_0, t]$ for every $u \in C_P([t_0, T]; \mathbb{R}^n)$ and S sat-
isfies the Lipschitz condition, i.e., there is a
constant $k > 0$ such that
 $||S(t, y_1) - S(t, y_2)|| \le k \sup_{t_0 \le s \le t} ||y_1(s) - y_2(s)||$,
for all $t \in [t_0, T], y_1, y_2 \in C_P([t_0, T]; \mathbb{R$ and S sat-

, there is a
 $(s) - y_2(s)$,
 $[t_0, T]; \mathbb{R}^n$.
 y , (3.11) [t_0, t] for every $u \in C_P([t_0, T]; \mathbb{R}^n)$ and S sat-
isfies the Lipschitz condition, i.e., there is a
constant $k > 0$ such that
 $||S(t, y_1) - S(t, y_2)|| \le k \sup_{t_0 \le s \le t} ||y_1(s) - y_2(s)||$,
for all $t \in [t_0, T]$, $y_1, y_2 \in C_P([t_0, T];$ $\begin{aligned}\n\text{(e0, c)} \text{ for every } a \in CP(\text{[e0, 1]}, \text{[s, 1]}, \text{[s, 1]}) \text{ and } S \text{ such } \\ \text{isfies the Lipschitz condition, i.e., there is a} \text{ constant } k > 0 \text{ such that} \\ \|\text{S}(t, y_1) - \text{S}(t, y_2)\| &\leq k \sup_{t_0 \leq s \leq t} \|y_1(s) - y_2(s)\| \,, \\ \text{for all } t \in [t_0, T], \ y_1, y_2 \in C_P([t_0, T]; \mathbb{R}^n). \\ \text{Then the equation} \\ y^\Delta$

$$
||S(t, y_1) - S(t, y_2)|| \le k \sup_{t_0 \le s \le t} ||y_1(s) - y_2(s)||
$$

 $(I_t, T]$: \mathbb{R}^n .

$$
y^{\Delta} = (P^{\Delta} + P_{\sigma} G^{-1} \overline{B}) y + P_{\sigma} G^{-1} S(t, y), \quad (3.11)
$$

constant $k > 0$ such that
 $||S(t, y_1) - S(t, y_2)|| \le k \sup_{t_0 \le s \le t} ||y_1(s) - y_2(s)||$,

for all $t \in [t_0, T]$, $y_1, y_2 \in C_P([t_0, T]; \mathbb{R}^n)$.

Then the equation
 $y^{\Delta} = (P^{\Delta} + P_{\sigma}G^{-1}\overline{B})y + P_{\sigma}G^{-1}S(t, y)$, (3.11)

with the initial $||S(t, y_1) - S(t, y_2)|| \le k \sup_{t_0 \le s \le t} ||y_1(s) - y_2(s)||$,
for all $t \in [t_0, T]$, $y_1, y_2 \in C_P([t_0, T]; \mathbb{R}^n)$.
Then the equation
 $y^{\Delta} = (P^{\Delta} + P_{\sigma} G^{-1} \overline{B})y + P_{\sigma} G^{-1} S(t, y)$, (3.11)
with the initial condition $y(t_0) = P(t_0)x_0$ has
 a unique solution in $C_P([t_0,T];\mathbb{R}^n)$. More- $\| - y_2(s) \|$,
, T]; \mathbb{R}^n).
), (3.11)
 $t_0)x_0$ has
). More-
iat if $y(t)$
then for all $t \in [t_0, T]$, $y_1, y_2 \in C_P([t_0, T]; \mathbb{R}^n)$.

Then the equation
 $y^{\Delta} = (P^{\Delta} + P_{\sigma} G^{-1} \overline{B})y + P_{\sigma} G^{-1} S(t, y)$, (3.11)

with the initial condition $y(t_0) = P(t_0)x_0$ has

a unique solution in $C_P([t_0, T]; \mathbb{R}^n)$. M for all $t \in [t_0, T]$, $y_1, y_2 \in C_P([t_0, T]; \mathbb{R}^n)$.

Then the equation
 $y^{\Delta} = (P^{\Delta} + P_{\sigma} G^{-1} \overline{B}) y + P_{\sigma} G^{-1} S(t, y)$, (3.11)

with the initial condition $y(t_0) = P(t_0) x_0$ has

a unique solution in $C_P([t_0, T]; \mathbb{R}^n)$. $y^{\Delta} = (P^{\Delta} + P_{\sigma} G^{-1} B)y + P_{\sigma} G^{-1} S(t, y)$, (3.11)

with the initial condition $y(t_0) = P(t_0)x_0$ has

a unique solution in $C_P([t_0, T]; \mathbb{R}^n)$. More-

over, there exists a constant c such that if $y(t)$

and $z(t)$ are two so

$$
||y(t) - z(t)|| \le c ||y(t_0) - z(t_0)||, \qquad (3.12)
$$

condition $u(t_0) = P(t_0)x_0$. Then, we use the formula (3.9) to obtain the solution of (3.2) with the initial condition $y(t_0) = P(t_0)x_0$ has
a unique solution in $C_P([t_0, T]; \mathbb{R}^n)$. More-
over, there exists a constant c such that if $y(t)$
and $z(t)$ are two solutions of (3.11) then
 $||y(t) - z(t)|| \le c ||y(t_0) - z(t_0)||$, (3.1 with the initial condition $y(t_0) = F(t_0)x_0$ has
a unique solution in $C_P([t_0, T]; \mathbb{R}^n)$. More-
over, there exists a constant c such that if $y(t)$
and $z(t)$ are two solutions of (3.11) then
 $||y(t) - z(t)|| \le c ||y(t_0) - z(t_0)||$, (3.1 a unique solution in $C_P([t_0, 1], \mathbb{R})$. Inter-
over, there exists a constant c such that if $y(t)$
and $z(t)$ are two solutions of (3.11) then
 $||y(t) - z(t)|| \le c ||y(t_0) - z(t_0)||$, (3.12)
By using this lemma, we see that the equation as $||y(t) - z(t)|| \le c ||y(t_0) - z(t_0)||$, (3.12)

By using this lemma, we see that the equation

(3.10) has a unique solution $u(\cdot)$ with initial

condition $u(t_0) = P(t_0)x_0$. Then, we use the

formula (3.9) to obtain the solution of (3.2 y using this lemma, we see that the equation
3.10) has a unique solution $u(\cdot)$ with initial
ondition $u(t_0) = P(t_0)x_0$. Then, we use the
rmula (3.9) to obtain the solution of (3.2)
 $t = u(t) + v(t)$
= $(H^{-1}\hat{P}u)(t) + (H^{-1}TQ_{\sigma}G^{-1$ *for to all in terminal, we see that the equation*

(3.10) has a unique solution $u(\cdot)$ with initial

condition $u(t_0) = P(t_0)x_0$. Then, we use the

formula (3.9) to obtain the solution of (3.2)

as
 $x(t) = u(t) + v(t)$
 $= (H^{-1}\hat$

$$
x(t) = u(t) + v(t)
$$

= $(H^{-1}\hat{P}u)(t) + (H^{-1}TQ_{\sigma}G^{-1}f)(t),$ (3.13)

condition $u(t_0) = P(t_0)x_0$. Then, we use the
formula (3.9) to obtain the solution of (3.2)
as
 $x(t) = u(t) + v(t)$
 $= (H^{-1}\hat{P}u)(t) + (H^{-1}TQ_{\sigma}G^{-1}f)(t)$, (3.13)
for $t \ge t_0$. The proof is complete. \square
Remark 3.4. *i) Inspired* formula (3.9) to obtain the solution of (3.2)

as
 $x(t) = u(t) + v(t)$
 $= (H^{-1}\hat{P}u)(t) + (H^{-1}TQ_{\sigma}G^{-1}f)(t)$, (3.13)

for $t \ge t_0$. The proof is complete. \square
 Remark 3.4. *i) Inspired by the above decou-

pling procedure, w* as
 $u(t) = u(t) + v(t)$
 $= (H^{-1} \hat{P}u)(t) + (H^{-1} T Q_{\sigma} G^{-1} f)(t)$, (3.13)

for $t \ge t_0$. The proof is complete. \square
 Remark 3.4. *i) Inspired by the above decou-

pling procedure, we state the initial condition*
 $u(t_0) = P(t_0)x_0$ $= u(t) + v(t)$
 $(H^{-1}\hat{P}u)(t) + (H^{-1}TQ_{\sigma}G^{-1}f)(t)$, (3.13)
 $t \ge t_0$. The proof is complete.
 nark 3.4. *i) Inspired by the above decou-*
 g procedure, we state the initial condition
 $P(t_0)(x(t_0) - x_0) = 0$, $x_0 \in \mathbb{R}^$

$$
P(t_0)(x(t_0) - x_0) = 0, \ x_0 \in \mathbb{R}^n. \qquad (3.14)
$$

 $(f)(s)\Delta s$ pend on the We note that the condition (3.
 $+P_{\sigma}G^{-1}\int_{t_0}^{t}K(t,s)H^{-1}(\hat{P}u+TQ_{\sigma}G^{-1}f)(s)\Delta s$ pend on the chosen projector

for all $t \geq t_0$. (3.10) (3.10). Multiplying both sides

we now use the following lemma, its proof tentio $\hat{P}u + TQ_{\sigma}G^{-1}f)(s)\Delta s$ pend on the chosen

ii) Let $u(t)$ be the

(3.10) (3.10). Multiplying t

with Q_{σ} yields $Q_{\sigma}u$

ing lemma, its proof tention that Q^{Δ} =

by using Picard's ap-obtains

d usual procedures (s) Δs pend on the chosen project

ii) Let $u(t)$ be the solution

(3.10) (3.10). Multiplying both side

with Q_{σ} yields $Q_{\sigma}u^{\Delta} = Q_{\sigma}$

proof tention that $Q^{\Delta} = (Q^2)^{\Delta}$:

i's ap-obtains

ures. $(Qu)^{\Delta} = Q^{\Delta}$
 For $t \ge t_0$. The proof is complete. \Box
 Remark 3.4. *i) Inspired by the above decoupling procedure, we state the initial condition* $u(t_0) = P(t_0)x_0$, or equivalent to
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W **Remark 3.4.** *i)* Inspired by the above decou-
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pend on the chosen projector opertor $Q(t_0$ $u(t_0) = P(t_0)x_0$, or equivalent to
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We note that the condition (3.14) does not de-

pend on the chosen projector opertor $Q(t_0)$.

ii) Let $u(t)$ be the solution of the equatio tention that $Q^{\Delta} = (Q^2)^{\Delta} = Q^{\Delta}Q_{\sigma} + QQ^{\Delta}$ obtains We note that the condition (3.14) does not de-
pend on the chosen projector opertor $Q(t_0)$.
ii) Let $u(t)$ be the solution of the equation
(3.10). Multiplying both sides of this equation
with Q_{σ} yields $Q_{\sigma}u^{\Delta} = Q_{$ pend on the chosen projector opertor $Q(t_0)$.

ii) Let $u(t)$ be the solution of the equation

(3.10). Multiplying both sides of this equation

with Q_{σ} yields $Q_{\sigma}u^{\Delta} = Q_{\sigma}P^{\Delta}u$. Paying at-

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tention that $Q^{\Delta} = (Q^2)^{\Delta} = Q^{\Delta}Q_{\sigma} + QQ^{\Delta}$

obtains
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$$
(Qu)^{\Delta} = Q^{\Delta}Qu
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Im $P(t_0)$ remains in Im $P(t)$ for all $t > t_0$ Denote by $\Phi_0(\cdot, \cdot)$ the Cauch
 $x(t_0) \in Im P(t_0)$ then $x(t) \in Im P(t)$, for all (3.17), i.e., it is the solution
 $t \in \mathbb{T}_t$.

(i *Nguyen Thu Ha/Nol 10. No* 4_August 2024| p.5-16
 Im P(t₀) remains in *Im P(t)* for all $t > t_0$ Denote by $\Phi_0(\cdot, \cdot)$ the Cauc
 $x(t_0) \in Im P(t_0)$ then $x(t) \in Im P(t)$, for all (3.17), i.e., it is the solution
 $t \in \mathbb{T}_{t_$ $t \in \mathbb{T}_{t_0}.$. Solution *Nguyen Thu Ha*/Vol 10

Im $P(t_0)$ remains in Im $P(t)$ for all $t > t_0$
 $x(t_0) \in Im P(t_0)$ then $x(t) \in Im P(t)$, for all
 $t \in T_{t_0}$.

ii) Since $TQ_{\sigma}G^{-1}$ is independent of the choice

of Q, so is the operator H.
 Nguyen Thu Ha/Vol 10. No

Im $P(t_0)$ remains in Im $P(t)$ for all $t > t_0$ De
 $x(t_0) \in Im P(t_0)$ then $x(t) \in Im P(t)$, for all (3.
 $t \in \mathbb{T}_{t_0}$.

(i) Since $TQ_{\sigma}G^{-1}$ is independent of the choice Φ_{σ}^{ϵ}

of Q, so is th

Nguyen Thu Ha/Vol 10. No 4_August 2024| p.5-16
 $Im P(t_0)$ remains in $Im P(t)$ for all $t > t_0$ Denote by $\Phi_0(\cdot, \cdot)$ the Cauca
 $x(t_0) \in Im P(t_0)$ then $x(t) \in Im P(t)$, for all (3.17), i.e., it is the solution
 $i \in \mathbb{T}_{t_0}$.
 $ii)$ $Im P(t_0)$ remains in $Im P(t)$ for all $t > t_0$ Deno
 $x(t_0) \in Im P(t_0)$ then $x(t) \in Im P(t)$, for all (3.17)
 $t \in \mathbb{T}_{t_0}$.

(a) Since $TQ_{\sigma}G^{-1}$ is independent of the choice $\Phi_0^{\Delta}(t)$

of Q, so is the operator H.

(b) We not $Im P(t_0)$ remains in $Im P(t)$ for all $t > t_0$ Denote by $\Phi_0(\cdot, \cdot)$ the Cauck $x(t_0) \in Im P(t_0)$ then $x(t) \in Im P(t)$, for all (3.17), i.e., it is the solution $t \in \mathbb{T}_{t_0}$.

ii) Since $TQ_{\sigma}G^{-1}$ is independent of the choice Im $P(t_0)$ remains in Im $P(t)$ for all $t > t_0$ Denote by $\Phi_0(\cdot, \cdot)$ the $x(t_0) \in Im P(t_0)$ then $x(t) \in Im P(t)$, for all (3.17), i.e., it is the s
 $t \in \mathbb{T}_{t_0}$.

(a) Since $TQ_{\sigma}G^{-1}$ is independent of the choice $\Phi_0^{\Delta}($ ii) Since $TQ_{\sigma}G^{-1}$ is independent of the choice $\Phi_0^{\Delta}(t, s) = (P^{\Delta} + P_{\sigma}G^{-1})$
of Q, so is the operator H.
iii) We note that for every $T > t_0$, the $+P_{\sigma}G^{-1}(t) \int_{s}^{t} K(t, \tau)$
space $C_{Q_{\sigma}G^{-1}}([t_0, T]; \mathbb{R}^n)$ is (b) since Γ and Γ are considered by σ (c, S) = Γ + Γ _oG

of Q, so is the operator H.

iii) We note that for every $T > t_0$, the $+P_{\sigma}G^{-1}(t)\int_s^t K(t,\tau)(s)$

space $C_{Q_{\sigma}G^{-1}}([t_0,T];\mathbb{R}^n)$ is independent iii) We note that for every $T > t_0$, the $+P_{\sigma}G^{-1}(t) \int_s^t K(t,\tau)(s)$
space $C_{Q_{\sigma}G^{-1}}([t_0,T];\mathbb{R}^n)$ is independent of the choice Q_{σ} and it is invariant under the and $\Phi_0(s,s) = I$ for all
the operator H.
We now try

ace $C_{Q_{\sigma}}G^{-1}([t_0, T]; \mathbb{R}^n)$ is independed

e choice Q_{σ} and it is invariant unde

e operator H.

e now try to give the variation of cons

rmula for the solution $x(\cdot)$ of the equ

(2). In order to do that, first the term is the variant wider the

the variant under the and $\Phi_0(s, s)$ =

directly differed

that, first we consider $u(t_0) = P(t_0)x$

tation, i.e., $f = 0$
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 $u(t) = \Phi_0(t, t_0)x$
 $u(t) = \Phi_0(t, t_0)x$
 $u(t) = \Phi_0(t, t$ *Cherry* the variation of constants the variation constant
formula for the solution $x(\cdot)$ of the equation tion $u(\cdot)$ of (3.10) wit
(3.2). In order to do that, first we consider $u(t_0) = P(t_0)x_0$
the homogeneous equation, We now try to give the variation of constants the variation constants formula
formula for the solution $x(\cdot)$ of the equation tion $u(\cdot)$ of (3.10) with the ir
(3.2). In order to do that, first we consider $u(t_0) = P(t_0)x_0$ formula for the solution $x(\cdot)$ of the equation tion $u(3.2)$. In order to do that, first we consider $u(t_0)$ =
the homogeneous equation, i.e., $f = 0$
 $A_{\sigma}(Py)^{\Delta}(t) = \overline{B}y(t) + \int_{t_0}^t K(t, s)y(s) \Delta s$. (3.15)
 $B(f)$
Define the

$$
A_{\sigma}(Py)^{\Delta}(t) = \overline{B}y(t) + \int_{t_0}^{t} K(t, s)y(s) \Delta s. \quad (3.15)
$$

the homogeneous equation, i.e., $f = 0$
 $A_{\sigma}(Py)^{\Delta}(t) = \overline{B}y(t) + \int_{t_0}^t K(t, s)y(s) \Delta s.$ (3.15)
 $\begin{aligned}\n&\int_{t_0}^{\tau} + \int_{t_0}^{\tau} K(\tau, h)H^{-1}TQ_{\sigma}G^{-1}f(t) \end{aligned}$

Define the Cauchy matrix $\Phi(t, s), t \geq s \geq t_0$

generated by homoge

$$
A(t)\Phi^{\Delta}(t,s) = B(t)\Phi(t,s) + \int_{s}^{t} K(t,\tau)\Phi(\tau,s)\Delta\tau,
$$

 $A_{\sigma}(Py)^{\Delta}(t) = \overline{B}y(t) + \int_{t_0}^{t} K(t, s)y(s) \Delta s.$ (3.15)

Define the Cauchy matrix $\Phi(t, s), t \geq s \geq t_0$

generated by homogeneous system (3.15) as On the other hand, since solution of the equation
 $\Phi_0(t, t_0)P(t_0)x_0$ and by Define the Cauchy matrix $\Phi(t, s)$, $t \ge s \ge t_0$

generated by homogeneous system (3.15) as On the other hand

the solution of the equation
 $A(t)\Phi^{\Delta}(t, s) = B(t)\Phi(t, s) + \int_s^t K(t, \tau)\Phi(\tau, s)\Delta\tau$, $\Phi(t, s) = (H^{-1}\hat{P}\Phi_0(\cdot))$

and $P(s)(\Phi$ Define the Cauchy matrix $\Phi(t, s), t$ is
generated by homogeneous system (
the solution of the equation
 $A(t)\Phi^{\Delta}(t, s) = B(t)\Phi(t, s) + \int_{s}^{t} K(t, \tau)\Phi$
and $P(s) (\Phi(s, s) - I) = 0$. Then, we l
variation of constants formula for the
of (3.2

the solution of the equation

relation between $\Phi(t, s)$
 $A(t)\Phi^{\Delta}(t, s) = B(t)\Phi(t, s) + \int_{s}^{t} K(t, \tau)\Phi(\tau, s)\Delta\tau$, $\Phi(t, s) = (H^{-1}\hat{P}\Phi_{0})$

and $P(s) (\Phi(s, s) - I) = 0$. Then, we have the

variation of constants formula for the solution

and
$$
P(s) (\Phi(s, s) - I) = 0
$$
. Then, we have the
variation of constants formula for the solution
of (3.2)
of (3.2)
of (3.2)
function (3.2) with the initial condition from the equation
 $P(t_0)(x(t_0) - x_0) = 0$ can be expressed as
 $x(t) = \Phi(t, t_0)P(t_0)x_0 + \int_{t_0}^t \Phi(t, \tau)P_{\sigma}G^{-1}(\tau) \begin{cases} = \frac{t}{C} \pi e^{-\frac{t}{C}} \\ = \frac{t}{C$

$$
(7) + \int_{t_0}^{R(\tau, h)(H - I Q_{\sigma} G - J)(h)\Delta h})\Delta \tau
$$
 Definition
\n
$$
(H^{-1}TQ_{\sigma}G^{-1}f)(t), t \geq t_0.
$$
 (3.16) *dyn*
\n
$$
vof. A similar procedure to split the solu-\nin of the homogeneous equation (3.15) into\n
$$
= \overline{u}(\cdot) + \overline{v}(\cdot) \text{ obtains}
$$

\n
$$
\overline{u}^{\Delta}(t) = (P^{\Delta} + P_{\sigma} G^{-1} \overline{B}) \overline{u}(t)
$$
 it
\n
$$
+ P_{\sigma} G^{-1} \int_{t_0}^{t} K(t, s) (H^{-1} \hat{P} \overline{u})(s) \Delta s, (3.17)
$$
 *expo
\nexist*
\nAnd $y(t) = (H^{-1} \hat{P} \overline{u})(t).$ (3.18) $||\Phi(x, t)|$
$$

And
$$
y(t) = (H^{-1}\hat{P}\overline{u})(t)
$$
. (3.18)

No 4_August 2024| p.5-16
Denote by $\Phi_0(\cdot, \cdot)$ the Cauchy operator of
(3.17), i.e., it is the solution of the matrix
equation No 4_August 2024| p.5-16

Denote by $\Phi_0(\cdot, \cdot)$ the Cauchy operator of

(3.17), i.e., it is the solution of the matrix

equation
 $\Phi_0^{\Delta}(t,s) = (P^{\Delta} + P_{\sigma}G^{-1}\overline{B})\Phi_0(t,s)$ equation

Nguyen Thu Ha/Vol 10. No 4 August 2024 | p.5-16
\nin Im
$$
P(t)
$$
 for all $t > t_0$ Denote by $\Phi_0(\cdot, \cdot)$ the Cauchy operator of
\nthen $x(t) \in Im P(t)$, for all (3.17), i.e., it is the solution of the matrix
\nequation
\nis independent of the choice $\Phi_0^{\Delta}(t,s) = (P^{\Delta} + P_{\sigma}G^{-1}\overline{B})\Phi_0(t,s)$
\nerator H.
\n t for every $T > t_0$, the $+P_{\sigma}G^{-1}(t) \int_s^t K(t,\tau)(H^{-1}\hat{P}_{\sigma}\Phi_0(\cdot,s))(\tau)\Delta\tau$
\n $T]; \mathbb{R}^n$ is independent of
\n d it is invariant under the
\nand $\Phi_0(s, s) = I$ for all $t \ge s \ge t_0$. Then, by
\ndirectly differentiating both sides we obtain
\ne the variation of constants the variation constants formula for the solu-
\nolution $r(\cdot)$ of the equation $u(\cdot)$ of (3.10) with the initial condition

Both sides by Γ_0 (,) are equation of the matrix
(3.17), i.e., it is the solution of the matrix
equation
 $\Phi_0^{\Delta}(t,s) = (P^{\Delta} + P_{\sigma}G^{-1}\overline{B})\Phi_0(t,s)$
 $+ P_{\sigma}G^{-1}(t)\int_s^t K(t,\tau)(H^{-1}\hat{P}_{\sigma}\Phi_0(\cdot,s))(\tau)\Delta\tau$
and $\Phi_0(s,s) = I$ for a equation
 $\Phi_0^{\Delta}(t,s) = (P^{\Delta} + P_{\sigma}G^{-1}\overline{B})\Phi_0(t,s)$
 $+ P_{\sigma}G^{-1}(t)\int_s^t K(t,\tau)(H^{-1}\hat{P}_{\sigma}\Phi_0(\cdot,s))(\tau)\Delta\tau$

and $\Phi_0(s,s) = I$ for all $t \ge s \ge t_0$. Then, by

directly differentiating both sides we obtain

the variation constants $\Phi_0^{\Delta}(t,s) = (P^{\Delta} + P_{\sigma}G^{-1}\overline{B})\Phi_0(t,s)$
 $+ P_{\sigma}G^{-1}(t)\int_s^t K(t,\tau)(H^{-1}\hat{P}_{\sigma}\Phi_0(\cdot,s))(\tau)\Delta\tau$

and $\Phi_0(s,s) = I$ for all $t \ge s \ge t_0$. Then, by

directly differentiating both sides we obtain

the variation constants formula fo $\Phi_0^-(t,s) = (P^{\perp} + P_{\sigma}G^{-1}B)\Phi_0(t,s)$
+ $P_{\sigma}G^{-1}(t)\int_s^t K(t,\tau)(H^{-1}\hat{P}_{\sigma}\Phi_0(\cdot,s))($
and $\Phi_0(s,s) = I$ for all $t \ge s \ge t_0$. The
directly differentiating both sides we o
the variation constants formula for the
tion $u(\cdot)$ of (3. + $P_{\sigma}G^{-1}(t) \int_{s} K(t,\tau) (H^{-1}\hat{P}_{\sigma}\Phi_{0}(\cdot,s))(\tau) \Delta \tau$

and $\Phi_{0}(s, s) = I$ for all $t \geq s \geq t_{0}$. Then, by

lirectly differentiating both sides we obtain

he variation constants formula for the solu-

ion $u(\cdot)$ of (3.10) w ⁻¹ $\hat{P}_{\sigma}\Phi_0(\cdot, s)(\tau)\Delta\tau$
 $\geq s \geq t_0$. Then, by

th sides we obtain

rmula for the solu-

ne initial condition
 $\int_{t_0}^t \Phi_0(t, \tau) P_{\sigma} G^{-1} (f(\tau))$
 $\Phi_0(h)\Delta h \Delta\tau$. (3.19) $\Phi_0(s, s) = I$ for all $t \ge s \ge t_0$. Then, by
ttly differentiating both sides we obtain
variation constants formula for the solu-
 $u(\cdot)$ of (3.10) with the initial condition
 $= P(t_0)x_0$
 $= \Phi_0(t, t_0)P(t_0)x_0 + \int_{t_0}^t \Phi_0(t, \tau)P$

$$
u_0 = 0
$$
\nand

\nthe variation constants formula for the solution $u(·)$ of (3.10) with the initial condition $u(t_0) = P(t_0)x_0$

\n
$$
u(t) = \Phi_0(t, t_0)P(t_0)x_0 + \int_{t_0}^t \Phi_0(t, τ)P_σ G^{-1}(f(τ))
$$

\n
$$
+ \int_{t_0}^r \Gamma(\tau, h)H^{-1}TQ_σ G^{-1}f(h) \Delta h \Delta \tau. \quad (3.19)
$$

\nOn the other hand, since $\bar{u}(t) = \Phi_0(t, t_0)P(t_0)x_0$ and by (3.18) we have the relation between Φ(t, s) and Φ_0(t, s)

\nΦ(t, s) = (H⁻¹ P Φ θ_0(·, s)P(s))(t).

\n(3.20)

\nThus, by acting $H^{-1}P$ to both sides of (3.19)

\nand paying attention to the expression (3.13)

$$
\Phi(t,s) = (H^{-1}\hat{P}\Phi_0(\cdot,s)P(s))(t). \quad (3.20)
$$

generated by homogeneous system (3.15) as $\overline{\Phi}_0(t,t_0)P(t_0)x_0$ and by (3.1
the solution of the equation
 $A(t)\Phi^{\Delta}(t,s) = B(t)\Phi(t,s) + \int_s^t K(t,\tau)\Phi(\tau,s)\Delta\tau$, $\Phi(t,s) = (H^{-1}\hat{P}\Phi_0(\cdot,s)P$
and $P(s)(\Phi(s,s) - I) = 0$. Then, we have the Thus, by $A(t)\Phi^{\Delta}(t,s) = B(t)\Phi(t,s) + \int_{s}^{t} K(t,\tau)\Phi(\tau,s)\Delta\tau, \quad \Phi(t,s) = (H^{-1}\hat{P}\Phi_{0}(\cdot,s))$
and $P(s)(\Phi(s,s) - I) = 0$. Then, we have the Thus, by acting $H^{-1}\hat{P}$ to b
variation of constants formula for the solution and paying attention to th
of (0. Then, we have the Thus, by acting H^-

and paying attentio

it is seen that the ural with the initial cond

lution $x(·)$ of the 0 can be given by t

initial condition formula (3.16). The

be expressed as **Assumptio** $\int_{t_0}^{\tau} \int_{t_0}^{t_0} f(t, t_0) \Delta t \, dt = \int_{t_0}^{\tau} \int_{t_0}^{t_0} f(t, t_0) \Delta t \, dt$ (3.19)
On the other hand, since $\overline{u}(t) = \Phi_0(t, t_0) P(t_0) x_0$ and by (3.18) we have the
relation between $\Phi(t, s)$ and $\Phi_0(t, s)$
 $\Phi(t, s) = (H^{-1} \hat{$ $+ \int_{t_0}^{\tau} K(\tau, h) H^{-1} T Q_{\sigma} G^{-1} f(h) \Delta h \Delta \tau$. (3.19)
On the other hand, since $\overline{u}(t) =$
 $\Phi_0(t, t_0) P(t_0) x_0$ and by (3.18) we have the
relation between $\Phi(t, s)$ and $\Phi_0(t, s)$
 $\Phi(t, s) = (H^{-1} \hat{P} \Phi_0(\cdot, s) P(s)) (t)$. (3.20)
 ⁺ $\int_{t_0}^{R} (7, h)H^{-1}Q_{\sigma}G^{-} f(h)\Delta h/\Delta h/\Delta t$. (3.19)

On the other hand, since $\overline{u}(t) =$
 $\Phi_0(t, t_0)P(t_0)x_0$ and by (3.18) we have the

relation between $\Phi(t, s)$ and $\Phi_0(t, s)$
 $\Phi(t, s) = (H^{-1}\hat{P}\Phi_0(\cdot, s)P(s))(t)$. (3.20)
 On the other hand, since $\overline{u}(t) = \Phi_0(t, t_0) P(t_0) x_0$ and by (3.18) we have the relation between $\Phi(t, s)$ and $\Phi_0(t, s)$
 $\Phi(t, s) = (H^{-1} \hat{P} \Phi_0(\cdot, s) P(s))(t)$. (3.20)

Thus, by acting $H^{-1} \hat{P}$ to both sides of (3.19)

an on the botter hand, since $u(t)$ –
 $\Phi_0(t, t_0)P(t_0)x_0$ and by (3.18) we have the

relation between $\Phi(t, s)$ and $\Phi_0(t, s)$
 $\Phi(t, s) = (H^{-1}\hat{P}\Phi_0(\cdot, s)P(s))(t)$. (3.20)

Thus, by acting $H^{-1}\hat{P}$ to both sides of (3.19)

and $\Phi(t, t_0)$ $\Phi(t, s) = (H^{-1}\hat{P}\Phi_0(\cdot, s)P(s))(t)$. (3.20)

Thus, by acting $H^{-1}\hat{P}$ to both sides of (3.19)

and paying attention to the expression (3.13)

and paying attention to the expression (3.13)

it is seen that the uni $\Phi(t,s) = (H^{-1}\hat{P}\Phi_0(\cdot,s)P(s))(t)$. (3.20)
Thus, by acting $H^{-1}\hat{P}$ to both sides of (3.19)
and paying attention to the expression (3.13)
it is seen that the unique solution $x(\cdot)$ of (3.2)
with the initial condition $P(t_0)(x$ Thus, by acting $H^{-1} \hat{P}$ to both sides of (3.19)
and paying attention to the expression (3.13)
and paying attention to the expression (3.13)
it is seen that the unique solution $x(\cdot)$ of (3.2)
with the initial conditio Thus, by acting $H^{-1}\hat{P}$ to both sides
and paying attention to the expressi
it is seen that the unique solution $x(\cdot)$
with the initial condition $P(t_0)(x(t_0)$
can be given by the variation of \cdot
formula (3.16). The pr acting $H^{-1}\hat{P}$ to both sides of (3.19)
g attention to the expression (3.13)
hat the unique solution $x(\cdot)$ of (3.2)
nitial condition $P(t_0)(x(t_0) - x_0) =$
given by the variation of constants
3.16). The proof is complete.

tiable projector $Q_{\sigma}(\cdot)$ onto $Ker A(\cdot)$ such that
 $TQ_{\sigma}G^{-1}$ and $P = I - Q$ are bounded on $[t_0, \infty)$.
Definition 3.7. It is seen that the unique solution $x(\cdot)$ of (3.2)
with the initial condition $P(t_0)(x(t_0) - x_0) = 0$ can be given by the variation of constants
formula (3.16). The proof is complete. \Box
Assumption 3.6. There exists a

the initial condition $P(t_0)(x(t_0) - x_0) =$
be given by the variation of constants
la (3.16). The proof is complete. \Box
mption 3.6. There exists a differen-
projector $Q_{\sigma}(\cdot)$ onto Ker $A(\cdot)$ such that
 ζ^{-1} and $P =$ be given by the variation of constants
la (3.16). The proof is complete. \square
mption 3.6. There exists a differen-
projector $Q_{\sigma}(\cdot)$ onto KerA(\cdot) such that
 ζ^{-1} and $P = I - Q$ are bounded on
).
ition 3.7. i) The im la (3.16). The proot is complete. \Box
 mption 3.6. There exists a differen-

projector $Q_{\sigma}(\cdot)$ onto Ker A(\cdot) such that
 t^{-1} and $P = I - Q$ are bounded on

).
 intion 3.7. i) The implicit integro-

dynamic equat **Example 13.6.** There exists a differen-

jector $Q_{\sigma}(\cdot)$ onto Ker $A(\cdot)$ such that

and $P = I - Q$ are bounded on
 on 3.7. i) The implicit integro-

amic equation (3.15) is uniformly

le if and only if there exists a po $Q_{\sigma}G^{-1}$ and $P = I - Q$ are bounded on
 (∞) .
 \in **efinition 3.7.** i) The implicit integro-

dynamic equation (3.15) is uniformly

stable if and only if there exists a posi-

tive number $M_0 > 0$ such that
 $\|\Phi(t, s)\| \$ tegro - equation (3.15) is said to be ^ω-exponentially stable if and only if there

$$
\|\Phi(t,s)\| \le M_0, \ t \ge s. \tag{3.21}
$$

dynamic equation (3.15) is uniformly
stable if and only if there exists a posi-
tive number $M_0 > 0$ such that
 $\|\Phi(t, s)\| \leq M_0, t \geq s.$ (3.21)
Let ω is regressive projective. The in-
tegro - equation (3.15) is said to b stable if and only if there exists a posi-
tive number $M_0 > 0$ such that
 $\|\Phi(t, s)\| \leq M_0, t \geq s.$ (3.21)
Let ω is regressive projective. The in-
tegro - equation (3.15) is said to be ω -
exponentially stable if and o

$$
\|\Phi(t,s)\| \le Me_{\ominus\omega}(t,s), \ t \ge s. \tag{3.22}
$$

Nguyen Thu HaNol 10. No 4_August 2024| p.5-16
 4 Stability of implicit in-

to

tegro - dynamic equa-

tion under small pertur-Nguyen Thu Ha/Vol 10. No 4_August 2024| p.5-16
 Stability of implicit in- to
 tegro - dynamic equa- $u^{\Delta}(t) = (P^{\Delta} + P_{\sigma}G^{-1}\bar{I})$
 tion under small pertur- $(\hat{P}u + TQ_{\sigma}G^{-1}f)(s)\Delta s +$
 bations Nguyen Thu Ha/Vol 10. No 4_August 2024| p.5-1
 Stability of implicit in- to
 tegro - dynamic equa- $u^{\Delta}(t) = (P^{\Delta} + P_{\sigma}G^{-1}f)(s)$
 bations for $T \ge t \ge t_0$. And bations **4 Stability of implicit in-** to
 tegro - dynamic equa- $u^{\Delta}(t) = (P^{\Delta} + P_{\sigma}G^{-1}\overline{B})u(t)$
 tion under small pertur-
 ($\hat{P}u+TQ_{\sigma}G^{-1}f(s)\Delta s+P_{\sigma}G$
 bations

In this section, we consider the effect of small

In **4 Stability of implicit in-** to
 tegro - dynamic equa- $u^{\Delta}(t) = (P^{\Delta} + P_{\sigma}G^{-1}\overline{B})u(t)$
 tion under small pertur-
 bations

for $T \ge t \ge t_0$. And

In this section, we consider the effect of small
 $x(t) = u(t) + v(t) = (H \$ **Example 11 All Stability Of Implicit int-** to
 tegro - dynamic equa-
 tion under small pertur-

($\hat{P}u+TQ_{\sigma}G^{-1}f$)(s) $\Delta s+P_{\sigma}G^{-1}$
 bations

In this section, we consider the effect of small

In this section Stability of implicit in- to

tegro - dynamic equa-

tion under small pertur-

bations
 $\hat{P}_u + TQ_\sigma G^{-1}f)(s)\Delta s + P_\sigma G^{-1}$

bations
 $\text{In this section, we consider the effect of small}$
 $\text{In this section, we consider the effect of small}$
 $x(t) = u(t) + v(t) = (H - H^{-1}TQ_\sigma G^{-1}F)(t)$

plicit integro - equation (3.15). Suppose that
 tion under small per
bations
In this section, we consider the effect of nonlinear perturbations to the stability
plicit integro - equation (3.15). Suppo
for every $t \ge t_0$, the perturbed equati
the form
 $A_{\sigma}(t)x^{\Delta}(t)$ **bations** for $T \ge t \ge$

n this section, we consider the effect of small $x(t) =$

onlinear perturbations to the stability of im-

licit integro - equation (3.15). Suppose that

or every $t \ge t_0$, the perturbed equation has on, we consider the effect of small
 $x(t) = u$
 $y(t) = u$
 $y(t$

$$
A_{\sigma}(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^{t} K(t,s)x(s)\Delta s
$$

$$
+ F(t,x(t)), \quad t \in \mathbb{T}_{t_0}.
$$
(4.1)

Follow the state of H and the set of the state of t of H

for every $t \geq t_0$, the perturbed equation has for $T \geq t \geq t_0$. If

the form
 $A_{\sigma}(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^t K(t, s)x(s) \Delta s$ $\Gamma_u(x)(t) = (H^{-1}\hat{P} + F(t, x(t)), t \in \mathbb{T}_t$ For every $t \geq t_0$, the perturbed equation has for $T \geq t \geq t_0$. Fix $u(\cdot) \in$

the form consider the mapping Γ_u
 $A_{\sigma}(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^t K(t, s)x(s) \Delta s \quad \Gamma_u(x)(t) = (H^{-1}\hat{P}u)(t) + H^{-}$
 $+ F(t, x(t)), t \in \mathbb{T}_u$. (4.1) for $T \$ solution $A_{\sigma}(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^t K(t,s)x(s) \Delta s$
 $C([t_0,T];\mathbb{R}^n)$ defined by
 $A_{\sigma}(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_0}^t K(t,s)x(s) \Delta s$
 $\Gamma_u(x)(t) = (H^{-1}\hat{P}u)(t) + H^{-1}T$
 $+ F(t,x(t)), t \in \mathbb{T}_t$. (4.1) for $T \ge t \ge t_0$. It is easy to s

Assume solvability of $(A, t) = B(t)x(t) + \int_{t_0}^t K(t, s)x(s)ds$ If
 $+ F(t, x(t)), t \in \mathbb{T}_{t_0}.$ (4.1) for \mathbb{R}

Assume that $F(t, 0) = 0$, for all $t \ge t_0$, which

follows that the equation (4.1) has the trivial

solvability of (4.1).

Assumpt $A_{\sigma}(t)x^{\Delta}(t) = B(t)x(t) + \int_{t_{0}} K(t,s)x(s)\Delta s$ $\Gamma_{u}(x)(t) = (H^{-1}\hat{P}u)(t) + H^{-1}$
 $+ F(t,x(t)), t \in \mathbb{T}_{t_{0}}.$ (4.1) for $T \ge t \ge t_{0}$. It is easy to

Assume that $F(t,0) = 0$, for all $t \ge t_{0}$, which $\sup_{t_{0} \le t \le T} ||\Gamma_{u}(x)(t) - \Gamma_{u}(x')$

follo

tions $P_{\sigma}G^{-1}(t)F(t, x)$ and $TQ_{\sigma}G^{-1}(t)F(t, x)$
are Lipschitz in x with Lipschitz coefficient $F(t, x(t)), t \in \mathbb{T}_{t_0}.$ (4.1) for $T \ge t \ge t_0.$
 $\vdots F(t, 0) = 0$, for all $t \ge t_0$, which $\sup_{t_0 \le t \le T} ||\Gamma_u(x)||$

the equation (4.1) has the trivial $\le \overline{\gamma}_T ||$
 $\equiv 0$. First at all, we consider the for any x, x'

f (4.1). 4 F(t, x(t)), $t \in \mathbb{I}_{t_0}$. (4.1) for $I \ge t \ge t_0$. It is early assume that $F(t, 0) = 0$, for all $t \ge t_0$, which $\sup_{t_0 \le t \le T} ||\Gamma_u(x)(t) - \Gamma$ follows that the equation (4.1) has the trivial $\le \overline{\gamma}_T ||H^{-1}||$ solution $x(\cdot$ Assume that $F(t, 0) = 0$, for all $t \ge t_0$, which

follows that the equation (4.1) has the trivial

solution $x(\cdot) \equiv 0$. First at all, we consider the

for any $x, x' \in C$

solvability of (4.1).
 Assumption 4.1. For all t Solution $x(\cdot) = 0$. First at an, we consider the
solvability of (4.1).
Assumption 4.1. For all $t \ge t_0$, the func- Hence, by
tions $P_{\sigma}G^{-1}(t)F(t, x)$ and $TQ_{\sigma}G^{-1}(t)F(t, x)$ ists unique
are Lipschitz in x with Lipschitz Solvability of (4.1).
 Assumption 4.1. For all $t \ge t_0$, the func- Hence, b

tions $P_{\sigma}G^{-1}(t)F(t, x)$ and $TQ_{\sigma}G^{-1}(t)F(t, x)$ ists uniq

are Lipschitz in x with Lipschitz coefficient
 l_t and γ_t respectively. Suppos $\begin{aligned}\n & \overline{\gamma}_T \| H^{-1} \| < 1, \Gamma_u \text{ is} \\
 & t \geq t_0, \text{ the } func. \text{ Hence, by the fixed p} \\
 & \Gamma Q_{\sigma} G^{-1}(t) F(t, x) \text{ ists uniquely an } x^* \in \text{sschitz coefficient} \\
 & \text{use further that } l. \text{ Denote } x^* = g(u) \text{ we} \\
 & \mathbb{R}^n) \text{ with the norm} \\
 & \text{be a constant} \\
 & \text{the normal} \\
 & \text{where } g(u)(t) = (H^{-1} \hat{P} \text{ and$

that $||H^{-1}||$ mean that the norm of opera**on 4.1.** For all $t \ge t_0$, the func-
 $\text{Hence, by the fixed point}$
 $\$ tor H^{-1} in $C_{TQ_{\sigma}G^{-1}}([t_0,T];\mathbb{R}^n)$. By denoting $G^{-1}(t)F(t, x)$ and $TQ_{\sigma}G^{-1}(t)F(t, x)$ ists uniquel

chitz in x with Lipschitz coefficient

respectively. Suppose further that l.

The continuous functions.

We $C_{TQ_{\sigma}G^{-1}}([t_0, T]; \mathbb{R}^n)$ with the norm
 $\lim_{t \to 0} C([t_0$ $\int_{\sigma} G^{-1}(t) F(t, x)$ ists uniquely an $x^* \in C$
hitz coefficient
further that l.
B. Denote $x^* = g(u)$ we have with the norm
 $g(u)(t) = (H^{-1} \hat{P} u)$
and understand
orm of opera-
Further,
 $\lim_{t \to T} ||g(u)(t) - g(u')(t)|| \le$
we
 $\lim_{[t_0, T]} ||g$ are Lipschitz in x with Lipschitz coefficient
 l_t and γ_t respectively. Suppose further than

and γ are continuous functions.

We endow $C_{TQ_{\sigma}G^{-1}}([t_0, T]; \mathbb{R}^n)$ with the inherited from $C([t_0, T]; \mathbb{R}^n)$ and $t_0 \leq s \leq t$ *itz* in x with Lipschitz coefficient

espectively. Suppose further that l.

continuous functions. Denote $x^* = g$
 $C_{TQ_{\sigma}G^{-1}}([t_0, T]; \mathbb{R}^n)$ with the norm $g(u)(t) = (1 + H)$

rom $C([t_0, T]; \mathbb{R}^n)$ and understand H
 $\$ and γ . are continuous functions. Denote $x^* = g$

We endow $C_{TQ_{\sigma}G^{-1}}([t_0, T]; \mathbb{R}^n)$ with the norm $g(u)(t) = ($

inherited from $C([t_0, T]; \mathbb{R}^n)$ and understand $+ H$

that $||H^{-1}||$ mean that the norm of opera-

func We endow $C_{TQ_{\sigma}G^{-1}}([t_0, T]; \mathbb{R}^n)$ with the norm

inherited from $C([t_0, T]; \mathbb{R}^n)$ and understand $H^{-1}TQ_{\sigma}G^{-1}$

that $||H^{-1}||$ mean that the norm of opera-

tor H^{-1} in $C_{TQ_{\sigma}G^{-1}}([t_0, T]; \mathbb{R}^n)$. By denot om $C([t_0, T]; \mathbb{R}^n)$ and understand $C_{TQ_{\sigma}G^{-1}}([t_0, T]; \mathbb{R}^n)$. By denoting
 $C_{TQ_{\sigma}G^{-1}}([t_0, T]; \mathbb{R}^n)$. By denoting $\sup_{[t_0, T]} ||g(u)(t) - g(u')||$

2. Let $T > t_0$. If $\overline{\gamma}_T ||H^{-1}|| < 1$, $+\overline{\gamma}_T ||H^{-1}|| \sup_{[t_0, t_0]}$
 Further,

tor H^{-1} in $C_{TQ_{\sigma}G^{-1}}([t_0, T]; \mathbb{R}^n)$. By denoting
 $\overline{\gamma}_t = \sup_{t_0 \le s \le t} \gamma_s$ for $t \ge t_0$, we have
 $\sup_{[t_0, T]} ||g(u)(t) - g(u')(t)|| \le \beta_T \sup_{[t_0, T]}$

Lemma 4.2. Let $T > t_0$. If $\overline{\gamma}_T ||H^{-1}|| < 1$,
 $\qquad \qquad + \overline{\gamma$

Lemma 4.2. Let $T > t_0$. If $\overline{\gamma}_T ||H^{-1}|| < 1$,
then the equation (4.1) with the initial condition **Example 3**
 $\overline{\gamma}_t = \sup_{t_0 \leq s \leq t} \gamma_s$ for $t \geq t_0$, we have $\sup_{[t_0,T]} \sup_{[t_0,T]}$
 Lemma 4.2. Let $T > t_0$. If $\overline{\gamma}_T ||H^{-1}|| < 1$,

then the equation (4.1) with the initial condition
 $P(t_0)(x(t_0) - x_0) = 0$, (4.2) dedu **Lemma 4.2.** Let $T > t_0$. If $\overline{\gamma}_T ||H^{-1}|| < 1$,

then the equation (4.1) with the initial condi-

tion
 $P(t_0)(x(t_0) - x_0) = 0$, (4.2)

is solvable on $[t_0, T]$. Further, there exists a $\sup_{t_0 \le t \le T} ||g(u)(t) - g(u')(t)$

constant then the equation (4.1) with the initial condi-

tion
 $P(t_0)(x(t_0) - x_0) = 0$, (4.2) deduces

is solvable on [t₀, T]. Further, there exists a $\sup_{t_0 \le t \le T} ||g(u)(t) - g(u')(t)|| \le L_T$ soonstant M_T such that

This means that g is

$$
P(t_0)(x(t_0) - x_0) = 0, \qquad (4.2)
$$

tion
 $P(t_0)(x(t_0) - x_0) = 0,$ (4.2) deduces

is solvable on [t₀, T]. Further, there exists a $\sup_{t_0 \le t \le T} ||g(u)(t) - g(u) ||$

constant M_T such that
 $||x(t)|| \le M_T ||P(t_0)x(t_0)||$, for all $t_0 \le t \le T$ with the Lipschit
 $||x(t)|| \le M_T ||P(t_0$

$$
||x(t)|| \le M_T ||P(t_0)x(t_0)||
$$
, for all $t_0 \le t \le T$.

upen Thu Ha/Vol 10. No 4_August 2024 | p.5-16
\n**mplicit in-** to
\n**mic equal-**
$$
u^{\Delta}(t) = (P^{\Delta} + P_{\sigma}G^{-1}\overline{B})u(t) + P_{\sigma}G^{-1}\int_{t_0}^t K(t, s)H^{-1}
$$

\n**all pertur-** $(\hat{P}u + TQ_{\sigma}G^{-1}f)(s)\Delta s + P_{\sigma}G^{-1}F(t, x(t))$ (4.3)
\nfor $T \ge t \ge t_0$. And
\nthe effect of small $x(t) = u(t) + v(t) = (H^{-1}\hat{P}u)(t)$
\nthe stability of im + $(H^{-1}TQ_{\sigma}G^{-1}F(\cdot, x(\cdot)))(t)$,
\n15). Suppose that
\nbed equation has for $T \ge t \ge t_0$. Fix $u(\cdot) \in C_P([t_0, T]; \mathbb{R}^n)$ and
\nconsider the mapping $\Gamma_u : C([t_0, T]; \mathbb{R}^n) \rightarrow$
\n $C([t_0, T]; \mathbb{R}^n)$ defined by
\n t_b
\n $K(t, s)x(s)\Delta s$ $\Gamma_u(x)(t) = (H^{-1}\hat{P}u)(t) + H^{-1}TQ_{\sigma}G^{-1}F(\cdot, x(\cdot))(t)$
\n∈ \mathbb{T}_{t_0} . (4.1) for $T \ge t \ge t_0$. It is easy to see that
\n $\sup_{t \in [T_u(x)(t) - \Gamma_u(x')(t)]}$

for $T \ge t \ge t_0$. Fix $u(\cdot) \in C_P([t_0, T]; \mathbb{R}^n)$ and $(I_t, T] \mathbb{R}^n \rightarrow$ \rightarrow $C([t_0,T];\mathbb{R}^n)$ defined by $x(t) = u(t) + v(t) = (H^{-1}\hat{P}u)(t)$
 $+ (H^{-1}TQ_{\sigma}G^{-1}F(\cdot, x(\cdot)))(t),$

for $T \ge t \ge t_0$. Fix $u(\cdot) \in C_P([t_0, T]; \mathbb{R}^n)$ and

consider the mapping $\Gamma_u : C([t_0, T]; \mathbb{R}^n) \rightarrow$
 $C([t_0, T]; \mathbb{R}^n)$ defined by
 $\Gamma_u(x)(t) = (H^{-1}\hat{P}u)(t) + H^{-1}T$

$$
\Gamma_{\!u}(x)(t) = (H^{-1}\hat{P}u)(t) + H^{-1}TQ_{\sigma}G^{-1}F(\cdot, x(\cdot))(t)
$$

) for $T \ge t \ge t_0$. It is easy to see that

$$
\sup_{t_0 \le t \le T} \|\Gamma_u(x)(t) - \Gamma_u(x')(t)\|
$$

$$
\le \overline{\gamma}_T \, \|H^{-1}\| \sup_{t \le t \le T} \|x(t) - x'(t)\|,
$$

(4.1) for $T \ge t \ge t_0$. It is easy to se
 t_0 , which $\sup_{t_0 \le t \le T} ||\Gamma_u(x)(t) - \Gamma_u(x')(t)||$

the trivial $\le \overline{\gamma}_T ||H^{-1}|| \sup_{t_0 \le t \le T} ||x||$

asider the for any $x, x' \in C([t_0, T])$
 $\overline{\gamma}_T ||H^{-1}|| < 1$, Γ_u is a contra

the func- H we consider the

for any $x, x' \in C$ C $\overline{\gamma}_T ||H^{-1}|| < 1$, Γ_u is a con
 $\ge t_0$, the func-

Hence, by the fixed point to $Q_{\sigma}G^{-1}(t)F(t, x)$ ists uniquely an $x^* \in C([t_0$

chitz coefficient
 e further that l.

ns. Den ${}^{t_0 \leq t \leq T}_{C}$ $(I_t, T]: \mathbb{R}^n$). Since consider the mapping $1_u : C([t_0, 1]; \mathbb{R}^n) \rightarrow$
 $C([t_0, T]; \mathbb{R}^n)$ defined by
 $\Gamma_u(x)(t) = (H^{-1}\hat{P}u)(t) + H^{-1}TQ_{\sigma}G^{-1}F(\cdot, x(\cdot))(t)$

for $T \ge t \ge t_0$. It is easy to see that
 $\sup_{t_0 \le t \le T} ||\Gamma_u(x)(t) - \Gamma_u(x')(t)||$
 $\le \overline{\gamma}_T ||H^{-1}|| \$: $C([t_0, I]; \mathbb{R}^n) \rightarrow$
 ${}^1TQ_{\sigma}G^{-1}F(\cdot, x(\cdot))(t)$

o see that
 $)(t)\|\$
 $\sum_{\substack{\sigma,\tau \ \in \mathcal{T}_0,\ \tau'\colon \mathbb{R}^n}} \|x(t) - x'(t)\|,$
 $[t_0, T]; \mathbb{R}^n)$. Since

intractive mapping.

theorem, there ex-
 $\sum_{\substack{\sigma,\tau \in \mathcal{T}_1}}$. \mathbb{R}^n $\begin{aligned} \n\mathbb{F}(\cdot, x(\cdot))(t) \n\end{aligned}$
 $\begin{aligned} \n\mathbb{E}^{r}(t) \Vert, \quad t \leq \mathbb{E}^{r}(t) \Vert, \quad t \leq \mathbb{E}^{r}(t) \quad \text{where} \quad t \leq \mathbb{E}^{r}(t) \quad \text{where} \quad t \leq \mathbb{E}^{r}(t) \quad \text{and} \quad t \leq \mathbb{E}^{r}(t) \quad \text{for} \quad t \leq \mathbb{E}^{r}(t) \quad \text{for} \quad t \leq \mathbb{E}^{r}(t) \quad \text{for} \quad$ $\overline{\gamma}_T$ $\|H^{-1}\|$ < 1, Γ_u is a co ; \mathbb{R}^n) defined by
 $=(H^{-1}\hat{P}u)(t)+H^{-1}TQ_{\sigma}G^{-1}F(\cdot,x(\cdot))(t)$
 $t \ge t_0$. It is easy to see that
 $\|\Gamma_u(x)(t)-\Gamma_u(x')(t)\|$
 $\le \overline{\gamma}_T \left\|H^{-1}\right\| \sup_{t_0 \le t \le T} \|x(t)-x'(t)\|$,
 $x, x' \in C([t_0, T]; \mathbb{R}^n)$. Since
 $\| < 1$, Γ_u is a $\Gamma_u(x)(t) = (H^{-1}\hat{P} u)(t) + H^{-1}TQ_{\sigma}G^{-1}F(\cdot, x(\cdot))(t)$
for $T \ge t \ge t_0$. It is easy to see that
 $\sup_{t_0 \le t \le T} ||\Gamma_u(x)(t) - \Gamma_u(x')(t)||$
 $\le \overline{\gamma}_T ||H^{-1}|| \sup_{t_0 \le t \le T} ||x(t) - x'(t)||$,
for any $x, x' \in C([t_0, T]; \mathbb{R}^n)$. Since
 $\overline{\gamma}_T ||H^{-1}|| < 1$ for $T \ge t \ge t_0$. It is easy to see that
 $\sup_{t_0 \le t \le T} ||\Gamma_u(x)(t) - \Gamma_u(x')(t)||$
 $\le \overline{\gamma}_T ||H^{-1}|| \sup_{t_0 \le t \le T} ||x(t) - x'(t)||$,

for any $x, x' \in C$ ([t_0, T]; \mathbb{R}^n). Since
 $\overline{\gamma}_T ||H^{-1}|| < 1$, Γ_u is a contractive mapping.

He ^{*} ∈ $C([t_0, T]; \mathbb{R}^n)$ such that to see that
 $x')(t)$ ||

up || $x(t) - x'(t)$ ||,
 $\mathcal{H}_{t \leq T}^{t \leq T}$
 $\mathcal{H}([t_0, T]; \mathbb{R}^n)$. Since

contractive mapping.

it theorem, there ex-

[t_0, T]; \mathbb{R}^n) such that
 (x^*) . $\begin{align*}\nx'(t) \|, \\
\text{Since } \text{mapping.} \text{there exists} \xrightarrow{} \text{such that}\n\end{align*}$ $I^{-1}\begin{aligned} &\|\nabla - \Gamma_u(x')(t)\| \end{aligned}$
 $\in \begin{aligned} &\|\nabla^{-1}\| \sup_{t_0 \leq t \leq T} \|x(t) - x'(t)\|, \\ &\in \text{C}^t(\overline{[t_0,T]}; \mathbb{R}^n). \text{ Since } \\ &\text{and } \text{point theorem, there exists } t \in C([t_0,T]; \mathbb{R}^n) \text{ such that } \\ &\|\cdot\| = \Gamma_u(x^*). \end{aligned}$

we have $\Gamma^1 \hat{P}_u(t)$ $\leq \overline{\gamma}_T ||H^{-1}|| \sup_{t_0 \leq t \leq T} ||x(t) - x'(t)||,$
 $x, x' \in C([t_0, T]; \mathbb{R}^n)$. Since
 $| < 1$, Γ_u is a contractive mapping.
 γ the fixed point theorem, there ex-
 rely an $x^* \in C([t_0, T]; \mathbb{R}^n)$ such that
 $x^* = \Gamma_u(x$ $\begin{aligned}\n &\geq |T||T|| \sup_{t\in\mathcal{U}} \sup_{t\in\mathcal{U}} |x(t)-x(t)||, \\
 &\text{any} \quad x, x' \in C^{\text{log}(t)}([t_0, T]; \mathbb{R}^n). \quad \text{Since} \\
 &|H^{-1}|| < 1, \Gamma_u \text{ is a contractive mapping.} \\
 &\text{ice, by the fixed point theorem, there ex-
uniquely an } x^* \in C([t_0, T]; \mathbb{R}^n) \text{ such that} \\
 & x^* = \Gamma_u(x^*). \\
 &\text{note } x^* = g(u) \text{ we have} \\
 & g(u$ x, $x \in C([t_0, t], \mathbb{R})$. Since
 $\langle 1, \Gamma_u$ is a contractive mapping.

the fixed point theorem, there ex-
 $\exists y$ an $x^* \in C([t_0, T]; \mathbb{R}^n)$ such that
 $x^* = \Gamma_u(x^*)$.
 $= g(u)$ we have
 $= (H^{-1}\hat{P}u)(t)$
 $+ H^{-1}TQ_{\sigma}G^{-1}F(\cdot, g(u$ $\begin{align*}\n\mathbf{r} &\text{if } \mathbf{r} \text{ is the matrix } \mathbf{r}$

$$
x^* = \Gamma_u(x^*).
$$

Denote $x^* = q(u)$ we have

$$
g(u)(t) = (H^{-1}\hat{P} u)(t) + H^{-1}TQ_{\sigma}G^{-1}F(\cdot, g(u(\cdot)))(t).
$$

Further,

$$
g(u)(t) = (H^{-1}Pu)(t)
$$

+ $H^{-1}TQ_{\sigma}G^{-1}F(\cdot, g(u(\cdot)))(t).$
Further,
\n
$$
\sup_{[t_0,T]} ||g(u)(t)-g(u')(t)|| \leq \beta_T \sup_{[t_0,T]} ||u(t)-u'(t)||
$$

\n
$$
+ \overline{\gamma}_T ||H^{-1}|| \sup_{[t_0,T]} ||g(u)(t)-g(u')(t)||,
$$

\nwith $\beta_T = ||H^{-1}\hat{P}||$. Letting $L_T = \frac{\beta_T}{1-\overline{\gamma}_T ||H^{-1}||}$
\ndeduces
\n
$$
\sup_{t_0 \leq t \leq T} ||g(u)(t)-g(u')(t)|| \leq L_T \sup_{t_0 \leq t \leq T} ||u(t)-u'(t)||.
$$

\nThis means that g is Lipschitz continuous
\nwith the Lipschitz coefficient L_T . In partic-
\nular,

with $\beta_T = ||H^{-1}\hat{P}||$. Letting $L_T = \frac{\beta_T}{1-\overline{\gamma}_T||H^{-1}||}$
deduces

$$
\sup_{t_0 \le t \le T} \|g(u)(t) - g(u')(t)\| \le L_T \sup_{t_0 \le t \le T} \|u(t) - u'(t)\|.
$$

with $\beta_T = ||H^{-1}|| \sup_{[t_0,T]} ||g(u)(t) - g(u')(t)||$,
with $\beta_T = ||H^{-1}\hat{P}||$. Letting $L_T = \frac{\beta_T}{1-\overline{\gamma}_T||H^{-1}}$
deduces
 $\sup_{0 \le t \le T} ||g(u)(t) - g(u')(t)|| \le L_T \sup_{t_0 \le t \le T} ||u(t) - u'(t)||$.
This means that g is Lipschitz continuous
with the Lipschitz coe lar,

$$
\sup_{t_0 \le t \le T} \|g(u)(t)\| \le L_T \sup_{t_0 \le t \le T} \|u(t)\| \, . \quad (4.4)
$$

Substituting
$$
x = g(u)
$$
 into (4.3) obtains

\n
$$
u^{\Delta}(t) = (P^{\Delta} + P_{\sigma}G^{-1}\overline{B})u(t) + P_{\sigma}G^{-1}F(t, g(u))(t)
$$
\nNote that for any $T \geq t \geq t_0$, the function $P_{\sigma}G^{-1}F(t, g(u))(t)$ for all $t \geq t_0$, where $P_{\sigma}G^{-1}F(t, g(u)(s)\Delta s)$.

\nNote that for any $T \geq t \geq t_0$, the function $P_{\sigma}G^{-1}F(t, g(u)(t))$ is Lipschitz in u . By applying again Lemma 3.3, we can solve $u(\cdot) = N(t) = \int_{t_0}^{t_0} (l_{\tau} + K_1 \int_{t_0}^{\tau} \overline{g_s} || P_{\sigma}G(t) ||_2) dt$.

 $P_{\sigma}G^{-1}F(t, g(u)(t))$ is Lipschitz in ing $x = g(u)$ into (4.3) obtains
 $\partial^{\Delta} + P_{\sigma} G^{-1} \overline{B} u(t) + P_{\sigma} G^{-1} F(t, g(u))(t)$
 $P_{\sigma} G^{-1} \int_{t_0}^t K(t, s) g(u)(s) \Delta s.$ (4.5)

t for any $T \ge t \ge t_0$, the function
 $t, g(u)(t)$ is Lipschitz in u. By ap-

ain Lemma 3.3, we can solv (a) into (4.3) obtains $x(\cdot)$ of (4.1) with the initial
 \overline{B}) $u(t)+P_{\sigma}G^{-1}F(t, g(u))(t)$
 $t, s)g(u)(s)\Delta s$. (4.5) $||x(t)|| \leq M_2e^{M_2N(t)}||P(t)$
 $\geq t \geq t_0$, the function

is Lipschitz in u. By ap-

a 3.3, we can solve $u(\cdot)$ Substituting $x = g(u)$ into (4.3) obtains $x(\cdot)$ of (4.1) with the a
 $u^{\Delta}(t) = (P^{\Delta} + P_{\sigma}G^{-1}\overline{B})u(t) + P_{\sigma}G^{-1}F(t, g(u))(t)$
 $+ P_{\sigma}G^{-1}\int_{t_0}^t K(t, s)g(u)(s)\Delta s.$ (4.5) $||x(t)|| \leq M_2e^{M_2N}$

Note that for any $T \geq t \geq t_0$, the f For all $t \geq t$ or $\int_{t_0}^{t} f(t, g(u)) dt$ satisfies
 $u^{\Delta}(t) = (P^{\Delta} + B_G^{-1} \overline{B}) u(t) + B_G^{-1} F(t, g(u)) (t)$ satisfies
 $+ P_G G^{-1} \int_{t_0}^{t} K(t, s) g(u)(s) \Delta s$. (4.5) $\|x(t)\| \leq M_2 e^{M_2 N(t)} \|P(t_0)$

for all $t \geq t_0$, where

Note that for any $u^{\Delta}(t) = (P^{\Delta} + P_{\sigma}G^{-1}\overline{B})u(t) + P_{\sigma}G^{-1}F(t, g(u))(t)$
 $+ P_{\sigma}G^{-1}\int_{t_0}^t K(t, s)g(u)(s)\Delta s.$ (4.5) $||x(t)|| \leq M_2e^{M_2N(t)}||F$

Note that for any $T \geq t \geq t_0$, the function
 $P_{\sigma}G^{-1}F(t, g(u)(t))$ is Lipschitz in u. By ap-

plying ag by that for any $T \ge t \ge t_0$, the function
 $F(t, g(u)(t))$ is Lipschitz in u. By apagain Lemma 3.3, we can solve $u(\cdot)$ $N(t) = \int_0^t (t_0 + t_0) dt_0$
 0 . Then the solution of (4.1) is given
 $x(t) = g(u)(t), T \ge t \ge t_0.$ (4.6) with the $[t_0$ Note that for any $T \ge t \ge t_0$, the function
 $P_{\sigma}G^{-1}F(t, g(u)(t))$ is Lipschitz in u. By ap-

plying again Lemma 3.3, we can solve $u(\cdot)$ $N(t) =$

from (4.5) with the initial condition $u(t_0) =$
 $P(t_0)x_0$. Then the solution plying again Lemma 3.3, we can solve $u(\cdot)$ $N(t) = \int (l_{\tau} + K_1)$
from (4.5) with the initial condition $u(t_0) =$
 $P(t_0)x_0$. Then the solution of (4.1) is given
by
 $x(t) = g(u)(t), T \ge t \ge t_0$. (4.6) with the initial
functions of so

$$
x(t) = g(u)(t), \ T \ge t \ge t_0. \tag{4.6}
$$

$$
||u(t)|| \le c ||u(t_0)||
$$
, $T \ge t \ge t_0$.

$$
||x(t)|| \le M_T ||P(t_0)x(0)||, \quad T \ge t \ge t_0,
$$

 $x(t) = g(u)(t), T \ge t \ge t_0.$ (4.6) with the initial c

Further, by Lemma 3.3

Further, by Lemma 3.3
 $||u(t)|| \le c ||u(t_0)||$, $T \ge t \ge t_0.$ $||\Phi(t, s)|| \le$

Combining (4.4) and (4.6) obtains
 $||x(t)|| \le M_T ||P(t_0)x(0)||$, $T \ge t \ge t_0$, mula (4.7), it From Lemma 4.2, it follows that the solution
 $\|x(t)\| \leq c \|u(t_0)\|$, $T \geq t \geq t_0$.
 $\|\Phi(t, s)\| \leq M_0$,
 $\text{Therefore, from the vari-2}$
 $\|x(t)\| \leq M_T \|P(t_0)x(0)\|$, $T \geq t \geq t_0$, $\text{mula } (4.7)$, it follows that

where $M_T = cL_T$. The proof is com $||u(t)|| \le c ||u(t_0)||$, $T \ge t \ge t_0$.
 $||\Phi(t, s)|| \le M_0$, t

Combining (4.4) and (4.6) obtains
 $||x(t)|| \le M_T ||P(t_0)x(0)||$, $T \ge t \ge t_0$, mula (4.7), it follows that for

where $M_T = cL_T$. The proof is complete. \Box $||x(t)|| \le M_0 ||P(t_0)x_0||$

Fro Difference, from the v

difference of the value of the value of $||x(t)|| \le M_T ||P(t_0)x(0)||$, $T \ge t \ge t_0$, mula (4.7), it follows

where $M_T = cL_T$. The proof is complete. \Box $||x(t)|| \le M_0 ||P(t_0)$.

From Lemma 4.2, it follows that th $\overline{\gamma}_T \left\| H^{-1} \right\| < 1$ for all $T > 1$ ng (4.4) and (4.6) obtains
 $\leq M_T ||P(t_0)x(0)||$, $T \geq t \geq t_0$, mula (4.7), it follows the var
 $\left|T_T = cL_T$. The proof is complete. \Box $||x(t)|| \leq M_0 ||P(t_0)x_0||$

mma 4.2, it follows that the solution $+ ||H^{-1}TQ_{\sigma}G^{-1}F(\cdot, x)$
 Final Evapondicity of the based of $||x(t)|| \le M_T ||P(t_0)x(0)||$, $T \ge t \ge t_0$, mula (4.7), it follows that for
where $M_T = cL_T$. The proof is complete. \Box $||x(t)|| \le M_0 ||P(t_0)x_0||$
From Lemma 4.2, it follows that the solution $+ ||H^{-1}TQ_{$ where $M_T = cL_T$. The proof is complete. \Box ||
From Lemma 4.2, it follows that the solution
 $x(\cdot)$ of the equation (4.1) with the initial con-
dition $P(t_0)(x(t_0)-x_0) = 0$ exists on $[t_0, \infty)$ if
 $\overline{\gamma}_T ||H^{-1}|| < 1$ for all

From Lemma 4.2, it follows that the solution
\n
$$
x(\cdot)
$$
 of the equation (4.1) with the initial con-
\ndition $P(t_0)(x(t_0)-x_0) = 0$ exists on $[t_0,\infty)$ if
\n
$$
\overline{\gamma}_T ||H^{-1}|| < 1
$$
 for all $T > t_0$ and the constant-
\nvariation formulas (3.16) can be written as
\n
$$
x(t) = \Phi(t, t_0)P(t_0)x_0 + \int_{t_0}^t \Phi(t, \tau)P_{\sigma}G^{-1}(\tau) \text{ By virtue of the Lipsch-\n
$$
P_{\sigma}G^{-1}F(\cdot, x(\cdot)) \text{ and } Q_{\sigma}G^{-1}
$$
\n
$$
\left(F(\tau, x) + \int_{t_0}^T K(\tau, s)H^{-1}TQ_{\sigma}G^{-1}F(\cdot, x)(s)\Delta s\right)\Delta \tau_{\parallel x}(t) \parallel \leq M_0 \parallel P(t_0)x_0 \parallel + \parallel H
$$
\n
$$
+ H^{-1}TQ_{\sigma}G^{-1}F(t, x(t)), \quad t \geq t_0. \qquad (4.7)
$$
\nTo proceed, firstly, we consider the boundedness of solutions of the equation (3.15) under small nonlinear perturbations.
\n
$$
\times ||H^{-1}|| \overline{\gamma}_s \sup_{t_0 \leq s_1 \leq s} ||x(s_1)||
$$
\n**Theorem 4.3.** Assume that the assumptions
\n3.6, 4.1 hold, the solutions of (3.15) is uni-
$$

For f_{t_0}
+ $H^{-1}TQ_{\sigma}G^{-1}F(t, x(t)), t \geq t_0.$ (4.7)
To proceed, firstly, we consider the bounded-
ness of solutions of the equation (3.15) under
small nonlinear perturbations.
Theorem 4.3. Assume that the assumptions
3. $H^{-1}TQ_{\sigma}G^{-1}F(t, x(t)), t \geq t_0.$ (4.7)

To proceed, firstly, we consider the boundedness of solutions of the equation (3.15) under

small nonlinear perturbations.

Theorem 4.3. Assume that the assumptions ≤

3.6, 4.1 hold, acting on $C_{TQ_{\sigma}G^{-1}}([0,\infty),\mathbb{R}^n)$ with $||H^{-1}|| = K_1$. Then, if $L = 1 - K_1 \overline{\gamma}_{\infty} > 0$, we can For proceed, in the solution of the equation (3.15) under

small nonlinear perturbations.

Since $\mathcal{L}_{bc} = \sum_{s=1 \leq s} |x(s)|$
 Theorem 4.3. Assume that the assumptions $\leq M_0 \|P(t_0)x_0\| + K_1$

3.6, 4.1 hold, the solutions

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ains $x(\cdot)$ of (4.1) with the init

satisfies
 $t, g(u)(t)$ $||x(t)|| \leq M_2 e^{M_2 N(t)}$

(4.5) for all $t > t_2$, where No 4_August 2024| p.5-16
 $x(\cdot)$ of (4.1) with the initial condition (3.14)

satisfies
 $||x(t)|| < M_2 e^{M_2 N(t)} ||P(t_0)x_0||,$ (4.8) satisfies No 4_August 2024| p.5-16
 $x(\cdot)$ of (4.1) with the initial condition (3.14)

satisfies
 $||x(t)|| \le M_2 e^{M_2 N(t)} ||P(t_0)x_0||,$ (4.8)

for all $t \ge t_0$, where
 $N(t) = \int_0^t (l_{\tau} + K_1 \int_0^{\tau} ||P_{\sigma} G^{-1} K(\tau, s) Q(s) ||\Delta s) \Delta \tau.$

$$
||x(t)|| \le M_2 e^{M_2 N(t)} ||P(t_0)x_0||, \qquad (4.8)
$$

$$
x(\cdot) \text{ of } (4.1) \text{ with the initial condition } (3.14)
$$
\n
$$
satisfies
$$
\n
$$
||x(t)|| \le M_2 e^{M_2 N(t)} ||P(t_0)x_0||, \qquad (4.8)
$$
\n
$$
\text{for all } t \ge t_0, \text{ where}
$$
\n
$$
N(t) = \int_{t_0}^t \left(l_r + K_1 \int_{t_0}^{\tau} ||P_{\sigma} G^{-1} K(\tau, s) Q(s)|| \Delta s \right) \Delta \tau.
$$
\n
$$
\text{Proof. Firstly, we note that the condition}
$$

 $||x(t)|| \le M_2 e^{M_2 N(t)} ||P(t_0)x_0||,$ (4.8)

for all $t \ge t_0$, where
 $N(t) = \int_{t_0}^t (l_\tau + K_1 \int_{t_0}^{\tau} ||P_\sigma G^{-1} K(\tau, s) Q(s)||\Delta s) \Delta \tau.$
 Proof. Firstly, we note that the condition
 $L > 0$ implies that the solution $x(\cdot)$ of (4.1)

w for all $t \ge t_0$, where
 $N(t) = \int_{t_0}^t (l_\tau + K_1 \int_{t_0}^\tau ||P_\sigma G^{-1} K(\tau, s) Q(s)||\Delta s) \Delta \tau$.

Proof. Firstly, we note that the condition
 $L > 0$ implies that the solution $x(\cdot)$ of (4.1)

with the initial condition (3.14) exists o for all $t \ge t_0$, where
 $N(t) = \int_{t_0}^t \left(l_{\tau} + K_1 \int_{t_0}^{\tau} ||P_{\sigma} G^{-1} K(\tau, s) Q(s)||\Delta s \right) \Delta \tau$.
 Proof. Firstly, we note that the condition
 $L > 0$ implies that the solution $x(\cdot)$ of (4.1)

with the initial condition (3. $N(t) = \int_{t_0}^t (l_{\tau} + K_1 \int_{t_0}^{\tau} ||P_{\sigma} G^{-1} K(\tau, s) Q(s)||\Delta s) \Delta \tau.$
 Proof. Firstly, we note that the condition
 $L > 0$ implies that the solution $x(\cdot)$ of (4.1)

with the initial condition (3.14) exists on
 $[t_0, \infty)$ by $N(t) = \int_{t_0}^{t} (l_{\tau} + K_1 \int_{t_0}^{\tau} \overline{\gamma}_s || P_{\sigma} G^{-1} K(\tau, s) Q(s) || \Delta s) \Delta \tau.$
 Proof. Firstly, we note that the condition
 $L > 0$ implies that the solution $x(\cdot)$ of (4.1)

with the initial condition (3.14) exists on
 $[t_0,$ Proof. Firstly, we note that the condition
 $L > 0$ implies that the solution $x(\cdot)$ of (4.1)

with the initial condition (3.14) exists on
 $[t_0, \infty)$ by Lemma 4.2. The uniform stability

of solutions of (3.15) says $L > 0$ implies that the solution $x(\cdot)$ of (4.1)
with the initial condition (3.14) exists on $[t_0, \infty)$ by Lemma 4.2. The uniform stability
of solutions of (3.15) says that
 $\|\Phi(t, s)\| \leq M_0, \quad t \geq s \geq t_0.$
Therefore,

$$
\|\Phi(t,s)\| \le M_0, \quad t \ge s \ge t_0.
$$

of solutions of (3.15) says that
\n
$$
T \ge t \ge t_0
$$
.
\n $||\Phi(t, s)|| \le M_0, \quad t \ge s \ge t_0$.
\nobtains
\nTherefore, from the variation of constants for-
\n $||, \quad T \ge t \ge t_0$, $|\Phi(4.7),$ it follows that for all $t \ge t_0$
\nof is complete. \Box $||x(t)|| \le M_0 ||P(t_0)x_0||$
\n $||H^{-1}TQ_{\sigma}G^{-1}F(\cdot, x(\cdot))(t)||$
\nwith the initial con-
\n $|\Phi(t, \infty))$ if $-\int_{t_0}^t M_0 (||P_{\sigma}G^{-1}(\tau)F(\tau, x(\tau))|| \int_{t_0}^{\tau} ||P_{\sigma}G^{-1}(\tau) \rangle$
\n $|\Phi(t, \tau)P_{\sigma}G^{-1}(\tau) \quad \text{By virtue of the Lipschitz conditions of}$
\n $P_{\sigma}G^{-1}F(\cdot, x(\cdot))$ and $Q_{\sigma}G^{-1}F(\cdot, x(\cdot))$, we get
\n $G^{-1}F(\cdot, x)(s)\Delta s \Delta \tau$
\n $|\Phi(t, \tau)P_{\sigma}G^{-1}(\tau) \quad \text{By virtue of the Lipschitz conditions of}$
\n $P_{\sigma}G^{-1}F(\cdot, x(\cdot))$ and $Q_{\sigma}G^{-1}F(\cdot, x(\cdot))$, we get
\n $G^{-1}F(\cdot, x)(s)\Delta s \Delta \tau$
\n $|\Phi(t, \tau)| \le M_0 ||P(t_0)x_0|| + ||H^{-1}|| \overline{\gamma}_t \sup_{t_0 \le s \le t} |x(s)||$
\n $t \ge t_0$. (4.7)

By virtue of the Lipschitz conditions of

$$
\overline{\gamma}_{T}||H^{-1}|| < 1 \text{ for all } T > t_{0} \text{ and the constant-}
$$
\nvariation formulas (3.16) can be written as\n
$$
+ K(\tau, s)H^{-1}TQ_{\sigma}G^{-1}(s)F(\cdot, x(\cdot))(s)||\Delta s) \Delta \tau.
$$
\n
$$
x(t) = \Phi(t, t_{0})P(t_{0})x_{0} + \int_{t_{0}}^{t} \Phi(t, \tau)P_{\sigma}G^{-1}(\tau) \text{ By virtue of the Lipschitz conditions of}
$$
\n
$$
P_{\sigma}G^{-1}F(\cdot, x(\cdot)) \text{ and } Q_{\sigma}G^{-1}F(\cdot, x(\cdot)), \text{ we get}
$$
\n
$$
\left(\frac{F(\tau, x) + \int_{t_{0}}^{T} f(\tau, s)H^{-1}TQ_{\sigma}G^{-1}F(\cdot, x)(s)\Delta s)\Delta \tau}{t_{0}}||x(t)|| \leq M_{0}||P(t_{0})x_{0}||+||H^{-1}||\overline{\gamma}_{t} \sup_{t_{0}\leq s\leq t}||x(s)||
$$
\n
$$
+ H^{-1}TQ_{\sigma}G^{-1}F(t, x(t)), \quad t \geq t_{0}. \tag{4.7}
$$
\nTo proceed, firstly, we consider the bounded-
\nness of solutions of the equation (3.15) under small nonlinear perturbations.\n
$$
\times ||H^{-1}|| \overline{\gamma}_{s} \sup_{t_{0}\leq s\leq s\leq s} ||x(s_{1})||\Delta s \right) \Delta \tau
$$
\n**Theorem 4.3.** Assume that the assumptions\n
$$
\leq M_{0}||P(t_{0})x_{0}|| + K_{1}\overline{\gamma}_{\infty} \sup_{t_{0}\leq s\leq t} ||x(s)||
$$
\n3.6, 4.1 hold, the solutions of (3.15) is uni-
\nformly stable and H^{-1} is a bounded operator\n
$$
+ M_{0} \int_{t_{0}}^{t} (l_{\tau} \sup_{t} ||x(s)|| + K_{1} \int_{t_{0}\leq s\leq t}^{\tau} ||x(s)||
$$
\n
$$
\text{acting on } Cr_{Q_{\sigma}G^{-1}}((0, \infty), \mathbb{R}^{n}) \text{ with } ||H^{-1}|| =
$$
\n
$$
K_{1}. \text{ Then, if } L = 1
$$

Putting $M_2 = \frac{M_0}{I}$, we have *Nguyen Thu Ha/*Vol 10. No 4_Augus

, we have Next, we define
 $\begin{aligned}\n\mathcal{H}_2 \| P(t_0) x_0 \|_2^T \| P(t_0) x_0 \|_2^T \| \leq \mathcal{H}_2^T \| \mathcal{H}_2(\mathcal{G}) \leq \mathcal{H}_2^T \| \mathcal{H}_2(\mathcal{G}) \leq \mathcal{H}_2^T \| \mathcal{H}_2(\mathcal{G}) \leq \mathcal{H}_2^T \| \mathcal{H}_2(\mathcal{G}) \leq \mathcal{H}_2^$ $\sup_{s\leq t}||x(s)|| \leq M_2 ||P(t_0)x_0||$ $t_0 \leq s \leq t$ $+ M_2 \int_{t_0}^{t} \!\!\! \left(l_{\tau} \sup_{t_0 \leq s \leq \tau} \lVert x(s) \rVert + K_1 \int_{t_0}^{\tau}$ l_{τ} sup $||x(s)|| + K_1$ | $\sup_{t_0 \leq s \leq \tau} ||x(s)|| + K_1 \int_{t_0}^{\tau} \overline{\gamma}_s \times \qquad \qquad \frac{\text{We det}}{\widetilde{G}}$ and $||P_{\sigma}G^{-1}(\tau)K(\tau,s)Q(s)||$ s $M_2 = \frac{M_0}{L}$, we have Nex
 $\|x(s)\| \le M_2 \|P(t_0)x_0\|$ bilit
 $\lambda \mu C$
 $\int_{t_0}^{t} \left(l_{\tau} \sup_{t_0 \le s \le \tau} \|x(s)\| + K_1 \int_{t_0}^{\tau} \overline{\gamma}_s \times \begin{array}{ccc} \text{We} & \text{We} \\ \widetilde{G} \text{ a} & \text{we} \\ \text{(7)}K(\tau, s)Q(s) \| \sup_{t_0 \le s_1 \le s} \|x(s_1)\| \Delta s \right) \Delta \tau. \text{$ $\left\|\sup_{t_0\leq s_1\leq s}||x(s_1)||\Delta s\right\|$ Putting $M_2 = \frac{M_0}{L}$, we have Next, we consid

sup $||x(s)|| \le M_2 ||P(t_0)x_0||$
 $+ M_2 \int_{t_0}^t (l_{\tau} \sup_{t_0 \le s \le \tau} ||x(s)|| + K_1 \int_{t_0}^{\tau} \overline{\gamma}_s \times \overline{\tilde{G}}$ and $\tilde{K}(t, h)$ in
 $||P_{\sigma}G^{-1}(\tau)K(\tau, s)Q(s)|| \sup_{t_0 \le s_1 \le s} ||x(s_1)||\Delta s \Delta \tau$ $\label{eq:2.1} \begin{array}{ll} \sup_{t_0\leq s\leq t}\|x(s)\|\leq M_2\, \|P(t_0)x_0\| & \text{ibility of}\\ \lambda\mu Q_\sigma); \hat{R} \\ +M_2\int_{t_0}^t\!\!\! \Big(l_\tau \sup_{t_0\leq s\leq \tau}\|x(s)\| +K_1\int_{t_0}^\tau \overline{\gamma}_s\times & \text{We define}\\ \Vert P_\sigma G^{-1}(\tau)K(\tau,s)Q(s)\Vert \sup_{t_0\leq s_1\leq s}\|x(s_1)\|\Delta s\Big)\Delta \tau. & \text{neutrals}\\ \text{Following the generalized Gronwall-Bellman}\\ \$ For all $t \geq t_0$, where
 $\|P_{\sigma}G^{-1}(\tau)K(\tau,s)Q(s)\| \sup_{t_0 \leq s \leq s} \|x(s_1)\|^{\Delta}$

Following the generalized Gronwall-E

inequality in Lemma 2.6
 $\|x(t)\| \leq \sup_{t_0 \leq s \leq t} \|x(s)\| \leq M_2 \|P(t_0)x_0\| e_{N_1}$

for all $t \geq t_0$, where

$$
||x(t)|| \le \sup_{t_0 \le s \le t} ||x(s)|| \le M_2 ||P(t_0)x_0||e_{N_1(\cdot)}(t, t_0)
$$

Following the generalized Gronwall-Bellman inequality in Lemma 2.6
\n
$$
||x(t)|| \le \sup_{t_0 \le s \le t} ||x(s)|| \le M_2 ||P(t_0)x_0||e_{N_1(\cdot)}(t, t_0)
$$
\nfor all $t \ge t_0$, where
\n
$$
N_1(\tau) = l_\tau + \int_{t_0}^{\tau} K_1 \overline{\gamma}_s ||P_\sigma G^{-1}(\tau) K(\tau, s) Q(s)|| \Delta s.
$$
\nSince $N_1(\cdot)$ is positive,
\n
$$
e_{N_1(\cdot)}(t, t_0) \le \exp\left(\int_{t_0}^t N_1(\tau) \Delta \tau\right) \qquad \Rightarrow
$$

e^N1(·) (t, t0) [≤] exp ^t ^t⁰ ^N1(^τ)∆τ [≤] exp ^t t⁰ ^l^τ ⁺ ^K¹ ^τ t⁰ ^γs"PσG−¹ (τ) [×] ^K(τ, s)Q(s)"∆^s -∆τ . Thus, "x(t)" [≤] ^M2e^N(t)"P(t0)x0" for all ^t [≥] ^t0. The proof is complete. As a consequence of Theorem 4.3 we see that

 $\leq \exp\Big(\int_{t_0} (l_{\tau} + K_1 \int_{t_0} \overline{\gamma}_s || P_{\sigma} G^{-1}(\tau))$ with M defined in
 α positive number ω
 $\times K(\tau, s)Q(s)||\Delta s)\Delta \tau\Big)$.

Thus, $||x(t)|| \leq M_2 e^{N(t)} ||P(t_0)x_0||$ for all $t \geq$ Proof. Let ε_0 be a p
 t_0 . The proof is $\langle J_{t_0} \rangle = J_{t_0}$ a positive number ω_1 such to a positive number ω_1 such to equation (4.1) is ω_1 -exponent

Thus, $||x(t)|| \leq M_2 e^{N(t)} ||P(t_0)x_0||$ for all $t \geq$ Proof. Let ε_0 be a positive r
 t_0 . The proof $\times K(\tau, s)Q(s)||\Delta s)\Delta \tau$.

Thus, $||x(t)|| \leq M_2e^{N(t)}||P(t_0)x_0||$ for all $t \geq$
 t_0 . The proof is complete. \square

As a consequence of Theorem 4.3 we see that Γ
 Corollary 4.4. Assume that the assumptions

3.6, 4.1 hold, t $\begin{aligned} \mathcal{E}(\mathbf{z}) \Delta \tau \Big) \,. \end{aligned}$ is $\Delta \tau$).
 $\mathcal{E}(\mathbf{z}) = \mathcal{E}(\mathbf{z})$ and $\mathcal{E}(\mathbf{z}) =$ Thus, $||x(t)|| \leq M_2 e^{N(t)} ||P(t_0)x_0||$ for all $t \geq PrC$
 t_0 . The proof is complete. \Box δ +

As a consequence of Theorem 4.3 we see that pos
 Corollary 4.4. Assume that the assumptions

3.6, 4.1 hold, the solutions of *acting on* $C_{TQ_{\sigma}G^{-1}}([0, \infty), \mathbb{R}^n)$ *with* $||H^{-1}|| = K_1$. If $L = 1 - K_1 \overline{\gamma}_{\infty} > 0$ *and* Thus, $||x(t)|| \leq M_2 e^{N(t)}||P(t_0)x_0||$ for all $t \geq Proot$. Let ε_0 be a pos
 t_0 . The proof is complete.

As a consequence of Theorem 4.3 we see that positive number $T_0 >$
 Corollary 4.4. Assume that the assumptions

3.6 uence of Theorem 4.3 we see that positive number 5
 1.4. Assume that the assumptions

d, the solutions of (3.15) is uni-
 $t_t + \tilde{K}_1 \int_{t_0}^t \overline{\gamma}_h e_\lambda(t, t)$

e and H^{-1} is a bounded operator
 $r_{Q_\sigma G^{-1}}([0, \infty), \mathbb{R}^$

formly stable and
$$
H^{-1}
$$
 is a bounded operator
\nacting on $C_{TQ_{\sigma}G^{-1}}([0, \infty), \mathbb{R}^n)$ with $||H^{-1}|| =$
\n K_1 . If $L = 1 - K_1 \overline{\gamma}_{\infty} > 0$ and
\n
$$
N = \int_{t_0}^{\infty} (l_{\tau} + \int_{t_0}^{T} K_1 \overline{\gamma}_s || P_{\sigma}G^{-1}K(\tau, s)Q(s)||\Delta s) \Delta \tau
$$
 for all $t \geq T_0$. To simplify no
\nfor all $t \geq T_0$. To simplify the solution $x(t, s, x)$
\n $\langle \infty,$
\n $A_{\sigma}(t)(Px)^{\Delta}(t) = \overline{B}(t)x(t) +$
\nthen, the solution of the equation (4.1) is uni-
\n $formly stable in the sense that$
\n $||x(t)|| \leq M_3 || P(t_0)x_0 ||$, $t \geq t_0$,
\nfor a certain constant M_3 .
\n $y^{\Delta}(t) = e_{\lambda}(\sigma(t), s)x^{\Delta}(t)$ -

$$
||x(t)|| \le M_3 ||P(t_0)x_0||, \quad t \ge t_0,
$$

 \widetilde{G} and $\widetilde{K}(t, h)$ instead of G and $K(t, h)$. Then nential stability of solutions of the equation $\lambda \mu \mathcal{Q}_{\sigma}$, $K(t, h) = e_{\lambda}(v(t), s)I$

We define the operators \widetilde{H} as i
 \widetilde{G} and $\widetilde{K}(t, h)$ instead of G an

we have the following theore
 $\lambda \tau$. nential stability of solutions

(3.15) under small nonline No 4_August 2024| p.5-16
Next, we consider the robust exponential sta-
bility of (3.15) For any $\lambda > 0$, let $\widetilde{G} = G(I + \lambda \mu Q_{\sigma})$; $\widetilde{K}(t, h) = e_{\lambda}(\sigma(t), s)K(t, h)e_{\Theta\lambda}(h, s)$.
We define the operators \widetilde{H} as in (3.6) No 4_August 2024| p.5-16

Next, we consider the robust exponential sta-

bility of (3.15) For any $\lambda > 0$, let $\tilde{G} = G(I + \lambda \mu Q_{\sigma})$; $\tilde{K}(t, h) = e_{\lambda}(\sigma(t), s)K(t, h)e_{\Theta \lambda}(h, s)$.

We define the operators \tilde{H} as in (3.6) No 4_August 2024| p.5-16

Next, we consider the robust exponential sta-

bility of (3.15) For any $\lambda > 0$, let $\tilde{G} = G(I + \lambda \mu Q_{\sigma})$; $\tilde{K}(t, h) = e_{\lambda}(\sigma(t), s)K(t, h)e_{\Theta\lambda}(h, s)$.

We define the operators \tilde{H} as in (3.6 No 4_August 2024| p.5-16
Next, we consider the robust exponential stability of (3.15) For any $\lambda > 0$, let $\tilde{G} = G(I + \lambda \mu Q_{\sigma})$; $\tilde{K}(t, h) = e_{\lambda}(\sigma(t), s)K(t, h)e_{\Theta\lambda}(h, s)$.
We define the operators \tilde{H} as in (3.6) by us No 4_August 2024| p.5-16
Next, we consider the robust exponential stability of (3.15) For any $\lambda > 0$, let $\tilde{G} = G(I + \lambda \mu Q_{\sigma})$; $\tilde{K}(t, h) = e_{\lambda}(\sigma(t), s)K(t, h)e_{\Theta \lambda}(h, s)$.
We define the operators \tilde{H} as in (3.6) by us Next, we consider the robust exponential stability of (3.15) For any $\lambda > 0$, let $\tilde{G} = G(I + \lambda \mu Q_{\sigma})$; $\tilde{K}(t, h) = e_{\lambda}(\sigma(t), s)K(t, h)e_{\Theta\lambda}(h, s)$.
We define the operators \tilde{H} as in (3.6) by using \tilde{G} and $\tilde{K}(t$ Next, we consider the robust exponential stability of (3.15) For any $\lambda > 0$, let $\tilde{G} = G(I + \lambda \mu Q_{\sigma})$; $\tilde{K}(t, h) = e_{\lambda}(\sigma(t), s)K(t, h)e_{\Theta\lambda}(h, s)$.
We define the operators \tilde{H} as in (3.6) by using \tilde{G} and $\tilde{K}(t$ bility of (3.15) For any $\lambda > 0$, let $\widetilde{G} = G(I + \lambda \mu Q_{\sigma})$; $\widetilde{K}(t, h) = e_{\lambda}(\sigma(t), s)K(t, h)e_{\Theta\lambda}(h, s)$.
We define the operators \widetilde{H} as in (3.6) by using
 \widetilde{G} and $\widetilde{K}(t, h)$ instead of G and $K(t, h)$. Then
w

 $||x(s)|| \le M_2 ||P(t_0)x_0||e_{N_1(\cdot)}(t,t_0)$ $(0,\omega)$ and $\ominus \lambda \in \mathcal{R}^+$ such that \widetilde{H}^{-1} acts sup $||x(s_1)||\Delta s/\Delta \tau$. nential stability of sotics (3.15) under small not

d Gronwall-Bellman
 Theorem 4.5. If the exponentially stable
 $|P(t_0)x_0||e_{N_1(\cdot)}(t, t_0)$ (0, ω) and $\ominus \lambda \in \mathcal{T}$

continuously on C₁
 $\|\widet$ We define the operators H as in (3.6) by using
 \widetilde{G} and $\widetilde{K}(t, h)$ instead of G and $K(t, h)$. Then

we have the following theorem about expo-

nential stability of solutions of the equation

(3.15) under small contraction of the spectrum of G and $K(t, h)$ instead of G and $K(t, h)$. Then
we have the following theorem about expo-
nential stability of solutions of the equation
(3.15) under small nonlinear perturbations.
Theorem $\binom{n}{k}$ with $\|\widetilde{H}^{-1}\| = \widetilde{K}_1$ satisfying $\widetilde{L} = 1 - \widetilde{K}_1 \overline{\gamma}_{\infty} > 0$.
Suppose further that we have the following theorem about exponential stability of solutions of the equation

(3.15) under small nonlinear perturbations.
 Theorem 4.5. If the equation (3.15) is ω -

exponentially stable and there exists an **Theorem 4.5.** If the equation (3.15

exponentially stable and there exists
 $(0, \omega)$ and $\ominus \lambda \in \mathcal{R}^+$ such that \widetilde{H}

continuously on $C_{TQ_{\sigma}G^{-1}}([0, \infty), \mathbb{R}^n$
 $\|\widetilde{H}^{-1}\| = \widetilde{K}_1$ satisfying $\widetilde{L} =$ equation (3.15) is ω -

nd there exists an $\lambda \in$

such that \widetilde{H}^{-1} acts
 $\widetilde{g} \widetilde{L} = 1 - \widetilde{K}_1 \overline{\gamma}_{\infty} > 0$.
 $e_{\lambda}(\tau, h) || P_{\sigma} G^{-1}(\tau)$
 $\leq \frac{\lambda \widetilde{L}}{2M(1 + \lambda u^*)}$ (4.9)

$$
(0, \omega) \text{ and } \ominus \lambda \in \mathcal{R}^+ \text{ such that } \tilde{H}^{-1} \text{ acts}
$$

\n
$$
(0, \omega) \text{ and } \ominus \lambda \in \mathcal{R}^+ \text{ such that } \tilde{H}^{-1} \text{ acts}
$$

\n
$$
||\tilde{H}^{-1}|| = \tilde{K}_1 \text{ satisfying } \tilde{L} = 1 - \tilde{K}_1 \overline{\gamma}_{\infty} > 0.
$$

\n
$$
\text{Suppose further that}
$$

\n
$$
\limsup_{\tau \to \infty} \left(l_{\tau} + \tilde{K}_1 \int_{t_0}^{\tau} \overline{\gamma}_h e_{\lambda}(\tau, h) || P_{\sigma} G^{-1}(\tau) \right)
$$

\n
$$
\times K(\tau, h) Q(h) || \Delta h \right) \le \frac{\lambda \tilde{L}}{2M(1 + \lambda \mu^*)} \quad (4.9)
$$

\nwith M defined in (3.22). That is, there is
\na positive number ω_1 such that the perturbed
\nequation (4.1) is ω_1 -exponentially stable.
\nProof. Let ε_0 be a positive number such that
\n $\delta + \varepsilon_0 \le \frac{\lambda}{2(1 + \lambda \mu^*)}$. Then, from (4.9), there is a
\npositive number $T_0 > 0$ such that

 $\delta + \varepsilon_0 \leq \frac{\lambda}{2(1+\lambda\mu^*)}$. Then, from (4.9) $2(1+\lambda\mu^*)$. Then, Home Δh) $\leq \frac{\lambda \tilde{L}}{2M(1 + \lambda \mu^*)}$ (4.9)
 in (3.22). *That is, there is*
 er ω_1 such that the perturbed
 i ω_1 -exponentially stable.

a positive number such that

. Then, from (4.9), there is a
 $T_0 > 0$ suc \times K (τ , n) $Q(n)$ || Δn) $\leq \frac{1}{2M(1 + \lambda\mu^*)}$ (4.9)

with M defined in (3.22). That is, there is

a positive number ω_1 such that the perturbed

equation (4.1) is ω_1 -exponentially stable.

Proof. Let $\varepsilon_$

with M defined in (3.22). That is, there is
\na positive number
$$
\omega_1
$$
 such that the perturbed
\nequation (4.1) is ω_1 -exponentially stable.
\nProof. Let ε_0 be a positive number such that
\n $\delta + \varepsilon_0 \leq \frac{\lambda}{2(1+\lambda\mu^*)}$. Then, from (4.9), there is a
\npositive number $T_0 > 0$ such that
\n $l_t + \widetilde{K}_1 \int_{t_0}^t \overline{\gamma}_h e_\lambda(t, h) ||P_\sigma G^{-1}(t) K(t, h) Q(h) ||\Delta h$
\n $< \delta + \varepsilon_0 \leq \frac{\lambda}{2(1+\lambda\mu^*)}$, (4.10)
\nfor all $t \geq T_0$. To simplify notations we write
\n $x(t)$ for the solution $x(t, s, x_0)$.
\n $A_\sigma(t)(Px)^\Delta(t) = \overline{B}(t)x(t) + \int_{t_0}^t K(t, s)x(s) \Delta s$
\n $+ F(t, x(t))$.

$$
\langle \delta + \varepsilon_0 \leq \frac{\lambda}{2(1 + \lambda \mu^*)}, (4.10)
$$

for all $t \geq T_0$. To simplify notations we write
 $x(t)$ for the solution $x(t, s, x_0)$.

$$
A_{\sigma}(t)(Px)^{\Delta}(t) = \overline{B}(t)x(t) + \int_{t_0}^t K(t, s)x(s) \Delta s + F(t, x(t)).
$$

Let $y(t) = e_{\lambda}(t, s)x(t), t \geq s \geq t_0$. Since

$$
y^{\Delta}(t) = e_{\lambda}(\sigma(t), s)x^{\Delta}(t) + \lambda e_{\lambda}(t, s)x(t)
$$

Let
$$
y(t) = e_{\lambda}(t, s)x(t), t \ge s \ge t_0
$$
. Since

$$
y^{\Delta}(t) = e_{\lambda}(\sigma(t), s)x^{\Delta}(t) + \lambda e_{\lambda}(t, s)x(t)
$$

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\nit is easy to see that *y* satisfies the equation Further,
$$
P_{\sigma}\tilde{G}^{-1}\tilde{F}(t, \cdot)
$$
 and
\n
$$
A_{\sigma}(t)(Py)^{\Delta}(t) = A_{\sigma}(t)(e_{\lambda}(t, s)Px)^{\Delta}(t)
$$
\n
$$
= A_{\sigma}(t)(e_{\lambda}(\sigma(t), s)(Px)^{\Delta} + \lambda e_{\lambda}(t, s)Px)
$$
\n
$$
= e_{\lambda}(\sigma(t), s)(\overline{B}(t)x(t) + \int_{t_0}^t K(t, s)x(s)\Delta s - A_{\sigma}(t)(Pz)^{\Delta}(t) = \overline{\tilde{B}}(t)z(t) +
$$
\n
$$
+ F(t, x(t)) + \lambda e_{\lambda}(t, s)A_{\sigma}(t)Px
$$
\n
$$
= [(1 + \lambda \mu(t))\overline{B}(t) + \lambda A_{\sigma}(t)P(t)]y(t)
$$
\n
$$
= \overline{\tilde{B}}(t, h), t \geq h \geq s \text{ of (4.1)}
$$
\n
$$
+ \int_{s}^{t} e_{\lambda}(\sigma(t), s)K(t, h)e_{\Theta\lambda}(h, s)y(h)\Delta h
$$
\n
$$
= \overline{\tilde{B}}y(t) + \int_{s}^{t} \widetilde{K}(t, h)y(h)\Delta h + \widetilde{F}(t, y(t)),
$$
\nTherefore, for all $t \geq h \geq s$
\nfor all $t \geq s$ and
\n
$$
(4.11)
$$
\n
$$
\overline{\tilde{B}}(t) = (1 + \lambda \mu(t))\overline{B}(t) + \lambda A_{\sigma}(t)P(t),
$$
\nThis means that (4.12) is
\n
$$
\overline{\tilde{B}}(t) = (1 + \lambda \mu(t))\overline{B}(t) + \lambda A_{\sigma}(t)P(t),
$$
\nAnd the solution of (4.11)
\nThis means that (4.12) is
\n
$$
\widetilde{F}(t, h) = e_{\lambda}(\sigma(t), s)K(t, h)e_{\Theta\lambda}(h, s),
$$
\n
$$
\widetilde{F}(t, y(t)) = \tilde{C}(\sigma(t, s)
$$

+
$$
e_{\lambda}(\sigma(t), s)F(t, e_{\Theta\lambda}(t, s)y(t))
$$
 - $(\cdot, \cdot) \rightarrow (s, \cdot) \rightarrow$

$$
\widetilde{G} = A_{\sigma} - \left[(1 + \lambda \mu) \overline{B} + \lambda A_{\sigma} P \right] T Q_{\sigma}
$$

$$
= G - \lambda \mu B T Q_{\sigma} = G \left(I - \lambda \mu G^{-1} B T Q_{\sigma} \right)
$$

$$
= G \left(I + \lambda \mu Q_{\sigma} \right).
$$

 $\widetilde{F}(t, y(t)) = e_{\lambda}(\sigma(t), s)F(t, e_{\ominus\lambda}(t, s)y(t)).$

Since
 $\widetilde{G} = A_{\sigma} - \left[(1 + \lambda\mu)\overline{B} + \lambda A_{\sigma}P \right] T Q_{\sigma} + \widetilde{H}$
 $= G - \lambda\mu BT Q_{\sigma} = G\left(I - \lambda\mu G^{-1}BTQ_{\sigma}\right)$
 $= G(I + \lambda\mu Q_{\sigma}).$

We see that $(I + \lambda\mu Q_{\sigma})^{-1} = \left(P_{\sigma} + (1 + \lambda\mu)Q_{\sigma}\right)^$ $(\lambda \mu) Q_{\sigma}$)⁻¹ = $P_{\sigma} + \frac{1}{1+\lambda_{\sigma} Q_{\sigma}} Q_{\sigma}$, which im- $\tilde{G} = A_{\sigma} - \left[(1 + \lambda \mu) \overline{B} + \lambda A_{\sigma} P \right] T Q_{\sigma} + \tilde{H}^{-1} T Q_{\sigma}$
 $= G - \lambda \mu B T Q_{\sigma} = G (I - \lambda \mu G^{-1} B T Q_{\sigma})$
 $= G (I + \lambda \mu Q_{\sigma}).$

We see that $(I + \lambda \mu Q_{\sigma})^{-1} = (P_{\sigma} + (1 + \lambda \mu) Q_{\sigma})^{-1} = P_{\sigma} + \frac{1}{1 + \lambda \mu} Q_{\sigma}$, which im-

plies $\$ plies \widetilde{G} is invertible, and $\widetilde{G}^{-1} = (P_{\sigma} +$ $\frac{1}{\Lambda} Q_{\sigma}$ G^{-1} , it is seen that = $G - \lambda \mu BTQ_{\sigma} = G(I - \lambda \mu G^{-1} BTQ_{\sigma})$ || $y(t)$ || $\leq MP(s)y_0 + \hat{I}$

= $G(I + \lambda \mu Q_{\sigma})$.

We see that $(I + \lambda \mu Q_{\sigma})^{-1} = (P_{\sigma} + (1 + \lambda \mu) \int_s^t ((1 + \lambda \mu) Q_{\sigma})^{-1} = P_{\sigma} + \frac{1}{1 + \lambda \mu} Q_{\sigma}$, which im-

plies \tilde{G} is invertible, and \til $= G(I + \lambda \mu Q_{\sigma}).$

We see that $(I + \lambda \mu Q_{\sigma})^{-1} = (P_{\sigma} + (1 + \lambda \mu)Q_{\sigma})^{-1} = P_{\sigma} + \frac{1}{1 + \lambda \mu}Q_{\sigma}$, which im-

plies \tilde{G} is invertible, and $\tilde{G}^{-1} = (P_{\sigma} + \frac{1}{1 + \lambda \mu}Q_{\sigma})G^{-1}$, it is seen that the equation

(4.11) is + $\lambda \mu Q_{\sigma}$)⁻¹ = $(P_{\sigma} + (1 + + \frac{H}{\sigma})$
 $P_{\sigma} + \frac{1}{1 + \lambda \mu} Q_{\sigma}$, which im-

rtible, and $\tilde{G}^{-1} = (P_{\sigma} + \times \tilde{H})$

it is seen that the equation

Furthermore, By using Theo

it is $(1 + \lambda \mu) Q_{\sigma} \tilde{G}^{-1} = Q_{\sigma} G^{-1}$.

$$
P_{\sigma}\widetilde{G}^{-1} = P_{\sigma}G^{-1}; \ \ (1 + \lambda\mu)Q_{\sigma}\widetilde{G}^{-1} = Q_{\sigma}G^{-1}.
$$

Moreover,

$$
\frac{1}{1+\lambda\mu}Q_{\sigma}G^{-1}, \text{ it is seen that the equation}
$$
\n
$$
(4.11) \text{ is index-1. Furthermore,}
$$
\n
$$
P_{\sigma}\tilde{G}^{-1} = P_{\sigma}G^{-1}; \quad (1+\lambda\mu)Q_{\sigma}\tilde{G}^{-1} = Q_{\sigma}G^{-1}. \qquad ||y(t)|| \leq \widetilde{M}e_{\widetilde{N}(t)}
$$
\nMoreover,\nwhere\n
$$
P_{\sigma}\tilde{G}^{-1}\tilde{F}(t, y(t)) = e_{\lambda}(\sigma(t), s) \qquad \widetilde{N}(\tau, s) = (1+\lambda \times P_{\sigma}G^{-1}F(t, e_{\ominus\lambda}(t, s)y(t)),
$$
\n
$$
Q_{\sigma}\tilde{G}^{-1}\tilde{F}(t, \cdot) = e_{\lambda}(t, s)Q_{\sigma}G^{-1}F(t, e_{\ominus\lambda}(t, s)y(t)). \qquad \times ||P_{\sigma}G^{-1}F(t, s)||_{\mathcal{L}}.
$$

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It is easy to see that y satisfies the equation Further, $P_{\sigma}\widetilde{G}^{-1}\widetilde{F}(t, \cdot)$
 $A_{\sigma}(t)(Py)^{\Delta}(t) = A_{\sigma}(t)(e_{\lambda}(t, s)Px)^{\Delta}(t)$ are $(1 + \lambda \mu(t))l_t$ and
 $\mu(t)(e_{\lambda}(t, s)(x), \lambda(t, \$ No 4_August 2024| p.5-16
Further, $P_{\sigma}\widetilde{G}^{-1}\widetilde{F}(t, \cdot)$ and $T Q_{\sigma}\widetilde{G}^{-1}\widetilde{F}(t, \cdot)$
are $(1 + \lambda \mu(t))l_t$ and γ_t -Lipschitz, respec-
tively. Consider the corresponding homoge-
neous equation to (4.11) No 4_August 2024| p.5-16
Further, $P_{\sigma}\widetilde{G}^{-1}\widetilde{F}(t,\cdot)$ and $TQ_{\sigma}\widetilde{G}^{-1}\widetilde{F}(t,\cdot)$
are $(1 + \lambda\mu(t))l_t$ and γ_t -Lipschitz, resp
tively. Consider the corresponding homo
neous equation to (4.11) nd $TQ_{\sigma}\widetilde{G}^{-1}\widetilde{F}(t, \cdot)$ -Lipschitz, respec-
sponding homoge-No 4_August 2024| p.5-16

Further, $P_{\sigma}\widetilde{G}^{-1}\widetilde{F}(t, \cdot)$ and $TQ_{\sigma}\widetilde{G}^{-1}\widetilde{F}(t, \cdot)$

are $(1 + \lambda\mu(t))l_t$ and γ_t -Lipschitz, respec-

tively. Consider the corresponding homoge-

neous equation to (4.11)
 $A(t)(P_{$ No 4_August 2024| p.5-16

Further, $P_{\sigma}\widetilde{G}^{-1}\widetilde{F}(t,\cdot)$ and $TQ_{\sigma}\widetilde{G}^{-1}\widetilde{F}(t,\cdot)$

are $(1 + \lambda\mu(t))l_t$ and γ_t -Lipschitz, respec-

tively. Consider the corresponding homoge-

neous equation to (4.11)
 $A_{\sigma}(t)($ No 4_August 2024| p.5-16

Further, $P_{\sigma}\tilde{G}^{-1}\tilde{F}(t, \cdot)$ and $TQ_{\sigma}\tilde{G}^{-1}\tilde{F}(t, \cdot)$

are $(1 + \lambda\mu(t))l_t$ and γ_t -Lipschitz, respec-

tively. Consider the corresponding homoge-

neous equation to (4.11)
 $A_{\sigma}(t)(Pz$ Further, $P_{\sigma}G^{-1}F(t, \cdot)$ and $TQ_{\sigma}G^{-1}F(t, \cdot)$
are $(1 + \lambda \mu(t))l_t$ and γ_t -Lipschitz, respec-
tively. Consider the corresponding homoge-
neous equation to (4.11)
 $A_{\sigma}(t)(Pz)^{\Delta}(t) = \overline{\widetilde{B}}(t)z(t) + \int_s^t \widetilde{K}(t, h)z(h)\Delta h.$

are
$$
(1 + \lambda \mu(t))l_t
$$
 and γ_t -Lipschitz, respec-
tively. Consider the corresponding homogeneous equation to (4.11)
\n $A_{\sigma}(t)(Pz)^{\Delta}(t) = \overline{\widetilde{B}}(t)z(t) + \int_s^t \widetilde{K}(t, h)z(h)\Delta h.$
\n(4.12)
\nBy definition, the Cauchy operator
\n $\widetilde{\Phi}(t, h), t \geq h \geq s$ of (4.12) and $\Phi(t, h)$ of
\n(3.15) have a relation
\n $\widetilde{\Phi}(t, h) = e_{\lambda}(t, h)\Phi(t, h), t \geq h \geq s.$
\nTherefore, for all $t \geq h \geq s$

 $A_{\sigma}(t)(1 \leq t)$ ($t) = D(t)\geq(t) + \int_{s}^{t} K(t, h)z(h)\Delta h.$

By definition, the Cauchy operator
 $\widetilde{\Phi}(t, h), t \geq h \geq s$ of (4.12) and $\Phi(t, h)$ of

(3.15) have a relation
 $\widetilde{\Phi}(t, h) = e_{\lambda}(t, h)\Phi(t, h), t \geq h \geq s.$

Therefore, for all By definition, the Cauchy operator
 $\widetilde{\Phi}(t, h), t \geq h \geq s$ of (4.12) and $\Phi(t, h)$ of

(3.15) have a relation
 $\widetilde{\Phi}(t, h) = e_{\lambda}(t, h)\Phi(t, h), t \geq h \geq s$.

Therefore, for all $t \geq h \geq s$
 $\|\widetilde{\Phi}(t, h)\| = e_{\lambda}(t, h) \|\Phi(t, h)\| \leq Me_{(\lambda$ perator (t, h) of s .
 $(t, h) \leq M$.

stable.

d by $\widetilde{\Phi}(t, h), t \geq h \geq s$ of (4.12) and $\Phi(t, h)$ of

(3.15) have a relation
 $\widetilde{\Phi}(t, h) = e_{\lambda}(t, h)\Phi(t, h), t \geq h \geq s.$

Therefore, for all $t \geq h \geq s$
 $\|\widetilde{\Phi}(t, h)\| = e_{\lambda}(t, h) \|\Phi(t, h)\| \leq Me_{(\lambda \oplus \omega)}(t, h) \leq M.$

This means that (

$$
\widetilde{\Phi}(t,h) = e_{\lambda}(t,h)\Phi(t,h), \ \ t \ge h \ge s.
$$

$$
\|\widetilde{\Phi}(t,h)\| = e_{\lambda}(t,h) \|\Phi(t,h)\| \le Me_{(\lambda \ominus \omega)}(t,h) \le M.
$$

(a, (3.15) have a relation)
\n
$$
y(t)
$$
\n
$$
\tilde{\Phi}(t, h) = e_{\lambda}(t, h)\Phi(t, h), t \geq h \geq s.
$$
\n(a, (111) $\|\tilde{\Phi}(t, h)\| = e_{\lambda}(t, h)\Phi(t, h), t \geq h \geq s$
\n
$$
= e_{\lambda}(t, h)\Phi(t, h)\| \leq Me_{(\lambda \oplus \omega)}(t, h) \leq M.
$$
\nThis means that (4.12) is uniformly stable.
\n
$$
\tilde{B}(t) + \lambda A_{\sigma}(t)P(t), \text{ And the solution of (4.11) is expressed by}
$$
\n
$$
t, h) e_{\ominus \lambda}(h, s),
$$
\n
$$
t, e_{\ominus \lambda}(t, s)y(t)
$$
\n
$$
= \tilde{\Phi}(t, s)P(s)y_{0} + \int_{s}^{t} \tilde{\Phi}(t, \tau)P_{\sigma}\tilde{G}^{-1}(\tau)\Big(\tilde{F}(\tau, y(\tau)) + \int_{s}^{t} \tilde{K}(\tau, h)\tilde{H}^{-1}TQ_{\sigma}\tilde{G}^{-1}\tilde{F}(\tau, x(\cdot))(h)\Delta h\Big)\Delta \tau
$$
\n
$$
\lambda A_{\sigma}P \Big| TQ_{\sigma} + \tilde{H}^{-1}TQ_{\sigma}\tilde{G}^{-1}\tilde{F}(t, y(t)).
$$
\n
$$
I - \lambda \mu G^{-1}B TQ_{\sigma} \Big) \qquad ||y(t)|| \leq MP(s)y_{0} + \tilde{K}_{1}\bar{\gamma}_{\infty} \sup_{t_{0} \leq h \leq t} ||y(h)||
$$
\n
$$
\frac{1}{\lambda \mu}Q_{\sigma}, \text{ which } \lim_{t \to \infty} + \int_{s}^{t} \left((1 + \lambda \mu(\tau))I_{\tau}||y(\tau)|| + \int_{s}^{t} \|\tilde{G}^{-1}(\tau)\tilde{K}(\tau, h)Q(h)\|\| + \int_{s}^{t} \|\tilde{G}^{-1}(\tau)\tilde{K}(\tau, h)Q(h)\|\| + \int_{s}^{t} \|\tilde{G}^{-1}(\tau)\tilde{K}(\tau, h)Q(h)\|\| + \int_{s}^{t} \|\tilde{G}^{-1}(\tau
$$

$$
y(t) = \Psi(t, s)P(s)y_0 + \int_s \Psi(t, \tau)P_\sigma G^{-1}(\tau) (F(\tau, y(\tau)))
$$

\n
$$
+ \int_s^{\tau} \widetilde{K}(\tau, h) \widetilde{H}^{-1}TQ_\sigma \widetilde{G}^{-1} \widetilde{F}(\cdot, x(\cdot))(h) \Delta h) \Delta \tau
$$

\n
$$
\overline{\partial} + \lambda A_\sigma P \Big] TQ_\sigma + \widetilde{H}^{-1}TQ_\sigma \widetilde{G}^{-1} \widetilde{F}(t, y(t)).
$$

\n
$$
G(I - \lambda \mu G^{-1} B T Q_\sigma) \qquad ||y(t)|| \leq MP(s)y_0 + \widetilde{K}_1 \overline{\gamma}_{\infty} \sup_{t_0 \leq h \leq t} ||y(h)||
$$

\n
$$
Q_\sigma)^{-1} = (P_\sigma + (1 + \mu M \int_s^t ((1 + \lambda \mu(\tau))l_\tau ||y(\tau)||
$$

\n
$$
\frac{1}{1 + \lambda \mu} Q_\sigma, \text{ which im-}\nand \widetilde{G}^{-1} = (P_\sigma + \lambda \widetilde{K}_1 || \overline{\gamma}_h \sup_{t_0 \leq h_1 \leq h} ||x(h_1)|| \Delta h) \Delta \tau.
$$

\n
$$
\text{where}
$$

\nBy using Theorem 4.3, with $\widetilde{M} = \frac{M}{\widetilde{L}}$ we have
\n
$$
+ \lambda \mu) Q_\sigma \widetilde{G}^{-1} = Q_\sigma G^{-1}. \qquad ||y(t)|| \leq \widetilde{M} e_{\widetilde{N}(\cdot, s)}(t, s) ||P(s)x_0||, t \geq s.
$$

\nwhere

$$
\therefore \quad ||y(t)|| \le \widetilde{M}e_{\widetilde{N}(\cdot,s)}(t,s) ||P(s)x_0||, \ t \ge s.
$$

where

$$
\widetilde{K}_{1} \|\overline{\gamma}_{h} \sup_{t_{0} \leq h_{1} \leq h} \|x(h_{1})\| \Delta h \right) \Delta \tau.
$$

\n
$$
\widetilde{G}^{-1} = Q_{\sigma} G^{-1}. \qquad \|y(t)\| \leq \widetilde{M} e_{\widetilde{N}(\cdot,s)}(t,s) \|P(s)x_{0}\|, \ t \geq s.
$$

\nwhere
\n
$$
\widetilde{G}^{-1} = Q_{\sigma} G^{-1}. \qquad \|y(t)\| \leq \widetilde{M} e_{\widetilde{N}(\cdot,s)}(t,s) \|P(s)x_{0}\|, \ t \geq s.
$$

\nwhere
\n
$$
\widetilde{N}(\tau,s) = (1 + \lambda \mu(\tau)) \left(l_{\tau} + \widetilde{K}_{1} \int_{s}^{\tau} \overline{\gamma}_{h} e_{\lambda}(\tau,h) \right)
$$

\n
$$
t, e_{\ominus \lambda}(t,s)y(t)), \qquad \times \|P_{\sigma} G^{-1}(\tau) K(\tau,h) Q(h) \| \Delta h \right)
$$

• Consider three cases when $t \geq T_0 \geq s \geq t_0$.	A. S. Andrew Problem
• Consider three cases when $t \geq T_0 \geq s \geq t_0$.	A. S. Andreev
• From (4.10) we see that (2018). On the problems of t equations, $Russ$ equations, $Ross$ equations, <	

where

$$
\leq Me_{\Theta\lambda}(t, s)e_{\tilde{N}}(t, s) || P(s)x_0 ||
$$
\n
$$
= Me_{\Theta\lambda}(t, s)e_{\tilde{N}}(t, T_0)e_{\tilde{N}}(T_0, s) || P(s)x_0 ||
$$
\n
$$
\leq Me_{\Theta\lambda}(t, s)e_{\tilde{N}}(t, s)e_{\tilde{N}}(T_0, t_0) || P(s)x_0 ||
$$
\n
$$
= Me_{\tilde{N}\Theta\lambda}(t, s)e_{\tilde{N}}(T_0, t_0) || P(s)x_0 ||
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= Me_{\tilde{N}\Theta\lambda}(t, s)e_{\tilde{N}}(T_0, t_0) || P(s)x_0 ||
$$
\n
$$
= Me_{\tilde{N}\Theta\lambda}(t, s)e_{\tilde{N}}(T_0, t_0) || P(s)x_0 ||
$$
\n
$$
= \sum_{i=1}^{n} \sum_{j=1}^{n} \
$$

Thus, $||x(t)|| \le K_1 e_{-\lambda_1}(t,s) ||P(s)x_0||$, where $K_1 = M e_{\tilde{N}(\cdot)}(T_0, t_0).$
• In case $t > s > T \ge t_0$, using a similar

 $||x(t)|| \leq M ||x(s)||e_{-\lambda_1}(t,s).$

 $\begin{aligned}\n &= \frac{\lambda/2}{1+\mu(\tau)\lambda} \quad \text{equations: an intr}\n & \text{applications, Univ}\n &\leq \frac{-\lambda/2}{1+\mu(\tau)\lambda} \leq \frac{-\lambda/2}{1+\mu^*\lambda} := -\lambda_1.\n \end{aligned}$ Thus, $||x(t)|| \leq K_1e_{-\lambda_1}(t, s)||P(s)x_0||$, where $K_1 = M e_{\tilde{N}(\cdot)}(T_0, t_0).$

• In case $t > s > T \geq t_0$, using a similar argu $\leq \frac{1}{1 + \mu(\tau)\lambda} \leq \frac{1}{1 + \mu^* \lambda} := -\lambda_1.$

Thus, $||x(t)|| \leq K_1 e_{-\lambda_1}(t, s)||P(s)x_0||$, where
 $K_1 = M e_{\tilde{N}(\cdot)}(T_0, t_0).$

• In case $t > s > T \geq t_0$, using a similar argument as above we get $||x(t)|| \leq M ||x(s)||e_{-\lambda_1}(t, s).$

• Consider Thus, $||x(t)|| \leq K_1 e_{-\lambda_1}(t, s)||P(s)x_0||$, where
 $K_1 = M e_{\tilde{N}(\cdot)}(T_0, t_0).$

• In case $t > s > T \geq t_0$, using a similar argument as above we get $||x(t)|| \leq M ||x(s)||e_{-\lambda_1}(t, s).$

• Consider the remaining case $t_0 \leq s \leq t \leq$ (2016). Find θ , $K_1 = M e_{\tilde{N}(\cdot)}(T_0, t_0)$.

• In case $t > s > T \ge t_0$, using a similar argument as above we get $||x(t)|| \le M||x(s)||e_{-\lambda_1}(t, s)$. Du, N. H., Li

• Consider the remaining case $t_0 \le s \le t \le (2016)$. On star of linear singu $T \geq t_0$, using a similar $\begin{array}{ll}\nEquad \text{and} & \text{for } t_0 \neq t_0,\n\end{array}$

we get $|x(s)||e_{-\lambda_1}(t, s).$ Du, Math

ining case $t_0 \leq s \leq t \leq$ (2016)

ining case $t_0 \leq s \leq t \leq$ (2016)

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nave By virtue • In case $t > s > T \ge t_0$, using a similar argument as above we get Math. Soc., Properties above we get $||x(t)|| \le M ||x(s)||e_{-\lambda_1}(t, s)$. Du, N. H., Line Consider the remaining case $t_0 \le s \le t \le$ (2016). On star flow the positivity (2016

interview and from the set of line

fined above and from the of line

have By virtue of the pos-

mma 4.2 we get *Differ*

equat
 $\frac{1}{2}(t, s)e_{\Theta} \frac{1}{2}(t, s)||x(s)||$. Choi,
 $\frac{1}{2}(t, s)e_{\Theta} \frac{1}{2}(t, s)||x(s)||$. Choi,
 \frac • Consider the remaining case $t_0 \le s \le t \le$ (2016). On stal
 T_0 . With $\lambda_1 > 0$ defined above and from the

inequality (4.5), we have By virtue of the pos-

itivity of \tilde{L} and Lemma 4.2 we get
 $||x(t)|| \le M_{T_0}e_{\frac{\lambda$

$$
||x(t)|| \le M_{T_0} e_{\frac{\lambda}{2}}(t,s)e_{\ominus\frac{\lambda}{2}}(t,s)||x(s)||.
$$

 $\frac{\lambda}{2} = \frac{-\lambda/2}{1 + \mu(\tau) \frac{\lambda}{2}} \leq \frac{-\lambda/2}{1 + \mu^* \lambda} := -\lambda_1$. Thus $||x(t)|| \leq M_{T_0} e_{\frac{\lambda}{2}}(T_0, t_0) e_{-\lambda_1}(t, s) ||x(s)||.$

 $||x(t)|| \leq Ke_{\Theta \omega_1}(t,s)||x(s)||$ for all $t \geq s \geq t_0$, the inequality (4.5), we have By virtue of the pos-

it ivity of \tilde{L} and Lemma 4.2 we get
 $||x(t)|| \leq M_{T_0} e_{\frac{\lambda}{2}}(t, s) e_{\Theta_{\frac{\lambda}{2}}}(t, s) ||x(s)||$. Choi, S. K., ar

Since $\Theta_{\frac{\lambda}{2}}^{\lambda} = \frac{-\lambda/2}{1 + \mu(\tau) \frac{\lambda}{2}} \leq \frac{-\lambda/2}{1$ $e_{\frac{\lambda}{2}}(T_0,t_0)\}$. The in l $||x(t)|| \leq M_{T_0}e_{\frac{\lambda}{2}}(t, s)e_{\Theta_{\frac{\lambda}{2}}}(t, s)||x(s)||.$

Since $\Theta_{2}^{\lambda} = \frac{-\lambda/2}{1+\mu(\tau)\frac{\lambda}{2}} \leq \frac{-\lambda/2}{1+\mu^{*}\lambda} := -\lambda_1$. Thus
 $||x(t)|| \leq M_{T_0}e_{\frac{\lambda}{2}}(T_0, t_0)e_{-\lambda_1}(t, s)||x(s)||.$

Combining the above estimates yields
 $||x(t)|| \leq Ke_{\Theta$ buning the above estimates yields
 $\|\leq Ke_{\Theta\omega_1}(t,s)\|x(s)\|$ for all $t \geq s \geq t_0$,
 $\in K = \max\{M, K_1, M_{T_0}e_{\frac{\lambda}{2}}(T_0, t_0)\}$. The $\begin{array}{c}\n\text{ential equation} \\
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 $\begin{array}{c}\n\text{D.D. Thuar} \\
\text{D.D. Thuar} \\
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\text$ and $\|S\| \leq Ke_{\Theta\omega_1}(t,s) \|x(s)\|$ for all $t \geq s \geq t_0$,
 $\in K = \max\{M, K_1, M_{T_0}e_{\frac{\lambda}{2}}(T_0, t_0)\}$. The
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e $K = \max\{M, K_1, M_{T_0}e_{\frac{\lambda}{2}}(T_0, t_0)\}\)$. The in Mathes

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