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THE LIMIT CONDITIONS FOR TRAPPING OF MICROLENS MODULATED ACOUSTIC WAVES IN ACOUSTIC-ELASTIC MEDIUM

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Abstract:

In some recent studies, the microlens 2D array modulated by acoustic wave is firstly proposed and the capability to use it to design optical tweezers 2D array theoretically investigated, i.e., the dependence of its focal length and its numerical aperture on the thickness of the acoustic-elastic medium (AEM) is discussed. In this paper, the conditions for microparticle trapping as the maximum gradient force larger than the necessary force (> 0.01pN) and radius of trapping region satisfied the diffractive limit ($R_{trap}>\lambda$), are discussed to find out the suitable collection of parameters.

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CÁC ĐIỀU KIỆN GIỚI HẠN CỦA BẪY SỬ DỤNG VI THẤU KÍNH BIẾN ĐIỆU SÓNG ÂM TRONG MÔI TRƯỜNG ĐÀN HỒI ÂM

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Tóm tắt

Trong một số nghiên cứu gần đây, mảng vi thấu kính 2D được điều chế bằng sóng âm đã được đề xuất và khả năng sử dụng nó để thiết kế kìm quang mảng 2D đã được nghiên cứu về mặt lý thuyết, tức là sự phụ thuộc của tiêu cự và khẩu độ số của nó vào độ dày của môi trường đàn hồi âm (AEM) sẽ được thảo luận. Trong bài viết này, các điều kiện để bẫy các vi hạt như lực gradient cực đại lớn hơn lực yêu cầu (> 0,01pN) và bán kính vùng bẫy thỏa mãn giới hạn nhiễu xạ (Rtrap>λ), sẽ được thảo luận để tìm ra bộ thông số thích hợp..

1. INTRODUCTION

The optical tweezers arrays (OTA) to manipulate an assembly of nanoparticle in specimen are presented in a lot of previous works. For example, Dufresne *et al* have created the multiple optical tweezers from a single laser beam using the diffractive optical elements and set up the computer-generated holographic optical tweezers (HOT) arrays, which are interested by Wei-Wei; Curtis *et al* have simulated the dynamic of HOTs; Jesacher demonstrated the diffractive optical tweezers (DOT) based on Fresnel regime, which are improved by Feng in 2012 and Sow *et al* successfully achieved the multiple-spot optical tweezers created with microlens arrays fabricated by proton beam writing. All mentioned optical tweezers arrays operate based on the spatial modulation of single laser beam by optical elements. In other ways, basing on spatial or temporal control, Tanaka *et al* manipulated particles by optical tweezers combined with the intelligent control techniques and Pornsuwancharoen proposed the novel dynamic optical tweezer array using dark soliton control within a Add/Drop multiplexer. Recently, X. Ren *et al* investigated successfully optical tweezer array system based on 2D photonic crystal.

Up to now, the acousto-elastic modulation in AEM has been proposed to use for partial reflection of light (as beam splitter), and the Bragg cell have found numerous application in photonics. It is great, McLeod and Arnold have discussed about the tunable acoustic gradient index lens in liquid, that is a good idea to design the optical tweezer 2D array.

So, based on the acoustic-elastic effect, in previous our work, we have firstly proposed using two perpendicular ultrasonic waves for creating the 2D array of microlenses in AEM and investigated the capability to design the optical tweezers array with it. However, there is a question what are the values of principle parameters as the laser power, acoustic frequency and AEM thickness, etc., for which the optical tweezer 2D array with microlens modulated by acoustic waves satisfy the diffractive limit, and its gradient force is higher the certain necessary force. The answer of this question is the purpose of this article.

2. THEORETICAL REMARKS

The 2D distribution of refractive index in AEM (Fig.1a) can be created by two acoustic waves propagating perpendicularly each to other. The expression of refractive index in plane (X,Y) is given as follows:

$$n(x,y) = n - \Delta n_0 \left[cos(2\pi x/\Lambda) + cos(2\pi y/\Lambda) \right] \quad (1)$$

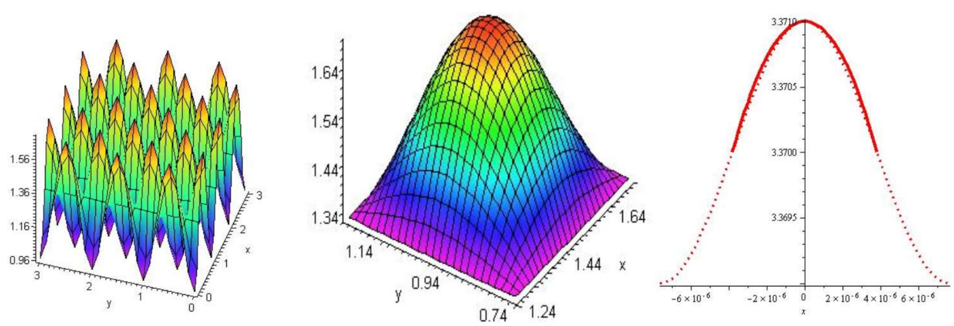
where n is the refractive index of AOM (Acousto optic materials) in the absence of sound,

$$\Delta n_0 = \sqrt{\frac{1}{2} M I_s} \quad (2)$$

is the refractive perturbation by acoustic wave,

$$M = \frac{\gamma^2 n^6}{g V_s^3} \quad (3)$$

is the figure of merit for the strength of the acoustic-elastic effect in AEM, I_s is the acoustic wave intensity, γ is a phenomenological coefficient known as the photoelastic constant (strain-acoustic constant) of AEM, V_s is the acoustic wave velocity in AEM, g is the mass density of AEM, and Λ is the acoustic wavelength.



a) **b)** **c)**

From Fig.1a, we can say that, the AEM becomes a 2D array of microlenses, one of them has the refractive index distribution of in area $\Lambda/2\times\Lambda/2$ as shown in Fig.2b. The expression of the refractive index distribution can be approximately rewritten as follows:

$$n(\rho) = N_0(1 - \frac{1}{2}\alpha^2\rho^2) \ , \quad (4)$$

where $-\Lambda/4 \le \rho = \sqrt{x^2+y^2} \le \Lambda/4$ is the radial distance from the microlens center (the center of area $\Lambda/2\times\Lambda/2$),

$$N_0 = n + 2\Delta n_0, \alpha^2 = \frac{64\Delta n_0}{N_0\Lambda^2}, \quad (5)$$

With the refractive index distribution as shown in Eq. 4 (see Fig.1c-solid curve), the AEM layer in a cycle with radius of $\Lambda/4$ is seen to be a GRIN (Gradient-Index) cylinder. Using Eq.4, the focal length of microlens is given as follows

$$f_s = \frac{1}{N_0\alpha^2 d} \quad (6)$$

where d is the thickness of GRIN cylinder. Using Eq.2, Eq.5 is rewritten as follows:

$$f_s = \frac{\Lambda^2}{64\Delta n_0 d} = \frac{V_s^2}{64\Delta n_0 d F_s^2} \qquad (7)$$

Considering behind AEM is a particle - embedding fluid with refractive index n_m, the numerical aperture is found out as:

$$NA = \frac{16\sqrt{MI_s}\,dn_m}{\sqrt{2}\Lambda} = \frac{16\sqrt{MI_s}\,dn_m F_s}{\sqrt{2}V_s}. \qquad (8)$$

The laser beam propagating through the microlens with diameter $D = \Lambda/2$ and focal length f_s will be focused and its intensity distribution follows the Airy pattern. Considering the laser beam is plane wave with constant average power P_0, after focusing by microlens, the plane wave beam is modified to Gaussian one (Fig.2) with intensity distribution given as follows:

$$I_L(\rho) = I_{L0} exp\left(-\frac{\rho^2}{2\rho_0^2}\right) \qquad (9)$$

where λ is the laser wavelength,

$$I_{L0} = \frac{P_0 A}{\lambda^2 f_s^2} = \frac{16\pi P_0 \Delta n\ d}{\lambda^2} \qquad (10)$$

is the peak intensity at the focus,

$$A = \frac{\pi\Lambda^2}{16} = \frac{\pi V_s^2}{16 F_s^2} \qquad (11)$$

is the area of the acoustic microlens, i.e. the cross-section of the plane laser beam, and

$$\rho_0 = \frac{0.9\lambda f_s}{\Lambda} = \frac{0.9\lambda V_s}{64\Delta n_0 d F_s} \qquad (12$$

is the diffraction limit.

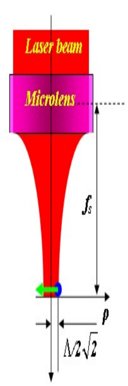


Fig. 2 Cartoon illustrating the construction of the optical tweezer using microlens.

Considering a particle with radius a and refractive index n_p is located around focus of microlens (Fig.2), i.e. at position $\left(f_s, -\Lambda/2\sqrt{2} \le \rho \le \Lambda/2\sqrt{2}\right)$, then on it acts an optical gradient force given as

$$\vec{F}_{gr,\rho}(\rho) = -\vec{\rho} n_0 \rho a^3 \frac{I_{L0}}{2\rho_0^2}\left(\frac{m^2-1}{m^2+2}\right) exp\left[-\left(\frac{\rho}{2\rho_0^2}\right)^2\right] \quad (13)$$

where $m = n_p/n_0$ is the refractive indexes of particle and fluid.

As shown in a lot of works, the most of the microparticles will be trapped if the optical force is about from 0.01pN to 100 pN, which is seen as a maximum force condition for trapping and the numerical aperture (NA) of optical system used for optical tweezer must be high ($NA \ge 1.1$). So, to use the microlens as an optical tweezer for microparticle, there are following conditions must satisfy: i) $NA \ge 1.1$; ii) The diffractive limit (or Gaussian beam waist), ρ_0 will also be the radius of trapping region, R_{trap}, so $\rho_0 > \lambda$ at least; iii) $F_{gr,max} > 0.01pN$, i.e. the maximum gradient force in trapping region is bigger than the thermal force of microparticle.

All proposed conditions depend on the principle parameters as: ultrasonic wave intensity, frequency, thickness of AEM and laser intensity, which are shown in Eqs.3÷13. These questions will be investigated in the next section.

3. THE LIMIT CONDITIONS FOR TRAPPING

As shown in Eqs. 3÷13, for the certain AEM with given figure of merit, M, consequently, the ultrasonic velocity V_s is given, the intensity distribution (9) and optical force (13) depend on the laser power, P_0, ultrasonic intensity, I_s, AEM's thickness, d, and ultrasonic frequency, F_s, except refractive, m and particle radius, a, on which optical force depends only.

We consider a glass particle with radius $a = 20nm$ and $n_p = 1.592$ embedded on fluid with $n_m = 1.326$, the AEM is the extra-dense flint glass (EDFG) with $M = 1.67\times10^{-14} m^2/W$, $d = 0.25cm$, $n = 1.92$ at optical wave length 1.15μm modulated by ultrasonic wave with $V_s \sim 3500m/s$ with considering that behind it the embedding fluid of trapped bio-molecule is water of $n_m = 1.326$.

Considering a case of ultrasonic intensity $I_s = 100W/cm^2$, then $\Delta n_0 \approx 5\times10^{-4}$, EDFG's thickness $d = 1.5\,mm$, laser power $P_0 = 100\,mW$. The gradient force distribution with two acoustic frequencies $F_s = 100\,MHz$ and $F_s = 150\,MHz$ are presented in Fig. 3 and the dependence of the maximum gradient force and the radius of trapping region on the acoustic frequency is presented in Fig.4.

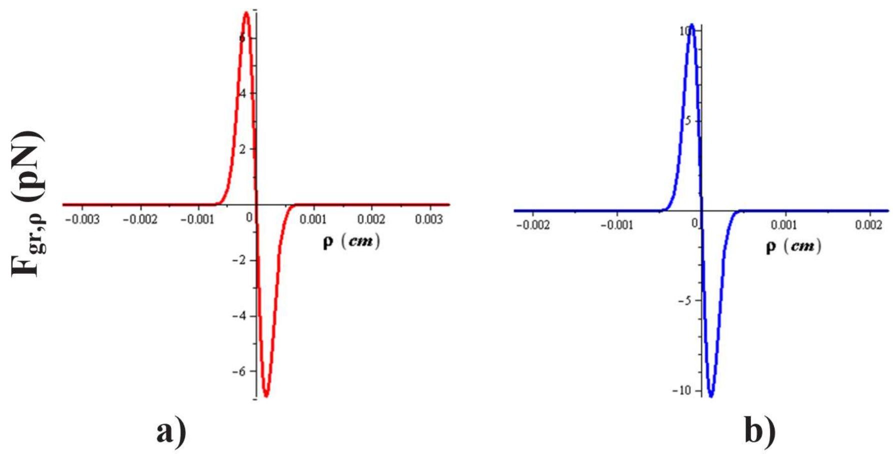


Fig. 3. Force distribution with two acoustic frequencies:100MHz (a) and

From Fig.3 and Fig.4, we can see that the maximum gradient force and the trapping region change with different frequency. The optical gradient force is enough to trap the microparticle. However, when the acoustic frequency increases, the gradient force increases too, but, instead the radius of trapping region (R_{trap} is distance between center of tweezer to point, where optical gradient force is maximum) decreases (see Fig.4). This phenomenon gives us to enhance the stability of the nanoparticle, but the trapping capability decreases, because that it is difficult to manipulate the microparticle locating far away from the center of trapping region.

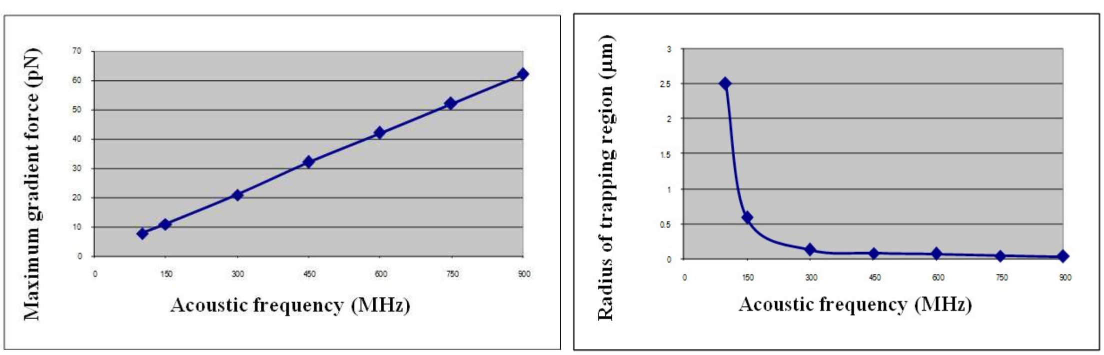
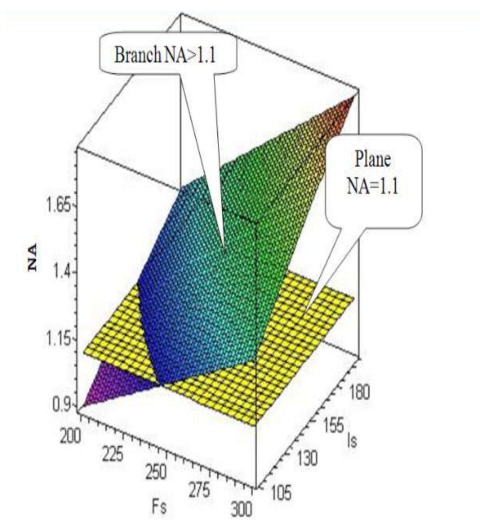


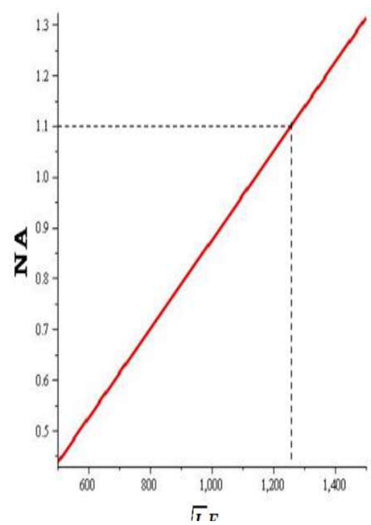
Fig. 4 The dependence of the maximum gradient force
(a) and radius of trapping region (b) on acoustic frequency.

Moreover, as shown in Fig.4b, if acoustic frequency is higher than 130 MHz, the radius of trapping region is shorter than 1μm. This radius is comparable to the laser wavelength. Consequently, this let us meet the diffraction-limit problem. So, we find out three limit conditions mentioned above.

For the first condition, we examine the NA of microlens appeared in extra-dense flint glass (EDFG). With parameters given above, the NA depends on two variable I_s and F_s is observed and presented in Fig.5. From Fig.5a, we can see that, in this case, the NA reaches 1.1 when intensity and frequency of ultrasonic wave are correspondingly each to other chosen, for example: $I_s = 105W/cm^2$ and $F_s = 240MHz$. As shown in Fig.5b, the first condition to use EDFG with d = 0.25 cm to create microlens, which operates as focusing system with $NA = 1.1$, will be satisfied if the collection of principle parameters must be chosen as:

$$\begin{cases} EDFG\ M = 1.67\times10^{-14} m^2/W, d = 0.25cm \\ Ultrasonic\ wave: \sqrt{I_s F_s} = 1250\sqrt{W/cm^2}\,MHz, V_s = 3500m/s \\ Embedding\ fluid: n = 1.326 \end{cases} \quad (14)$$





a) **b)**

Fig.5. The dependence of numerical aperture on acoustic wave intensity and frequency (a), and on product $\sqrt{I_s F_s}$ (b).

For the second and the third conditions, from Eq.13 we find out the expressions of maximum gradient force, $F_{gr,max}$ and relating radius of trapping region, R_{trap}. Radius of trapping region R_{trap} is equal to ρ_{max}, at which the derivation of gradient force is zero. From (13), we have:

$$\frac{dF_{gr,\rho}(\rho)}{d\rho} = -n_0\rho a^3 \frac{I_{L0}}{2\rho_0^2}\left(\frac{m^2-1}{m^2+2}\right) exp\left[-\left(\frac{\rho}{\sqrt{2}\rho_0^2}\right)^2\right]\left(1-\frac{\rho^2}{\rho_0^2}\right) \quad (15)$$

Let $\frac{dF_{gr,\rho}(\rho)}{d\rho}=0$, we have

$$\left|\rho_{max}\right| = \rho_0 = R_{trap}, \qquad (16)$$

and

$$F_{gr,max} = n_0 a^3 \frac{I_{L0}}{2\rho_0}\left(\frac{m^2-1}{m^2+2}\right)exp\left(-\frac{1}{2}\right) \qquad (17)$$

Eq. 16 means that the radius of trapping region is equal to diffraction limit. To avoid the diffraction- limit problem, we consider the radius of trapping region is equal to the laser wavelength at least. This means, using Eq.12, the following condition must be satisfied

$$\frac{0.9\lambda V_s}{32\Delta n_0 dF_s} \geq \lambda \ or \frac{0.9V_s}{32\Delta n_0 dF_s} \geq 1 \qquad (1$$

Eq.18 is to be seen as the first limit condition for trapping.

Using Eq.11, Eq.12 and Eq.17, and the condition $F_{gr,\rho} > 0.01\,pN$, we have the second condition for trapping as follows:

$$\frac{256\pi P_0 M I_s d^2}{\lambda^3}\frac{n_m a^3 F_s}{0.9V_s}\left(\frac{m^2-1}{m^2+2}\right)exp\left(-\frac{1}{2}\right) \geq 10^{-14}(N)$$
(19)

With parameters given in Coll.14, the dependence of ρ_0/λ on product $\sqrt{I_s}F_s$ is presented in Fig.6. From Fig.6, we have

$$\sqrt{I_s}F_s = 1350\sqrt{W/cm^2}MHz \Leftrightarrow \rho_0/\lambda = 1$$
, (20)

and ρ_0/λ increases when $\sqrt{I_s}F_s$ decreases. However, increasing of ρ_0/λ corresponds to decreasing of NA. So, the first and second conditions satisfy when

$$1250 < \sqrt{I_s}F_s < 1350\,. \qquad (21)$$

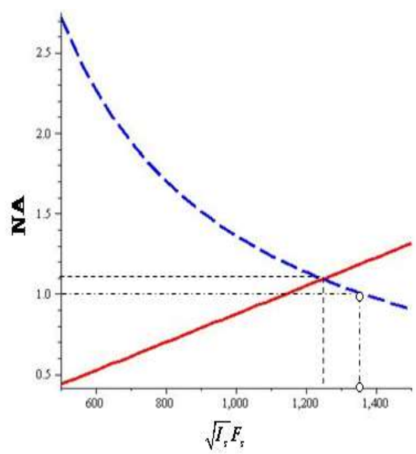
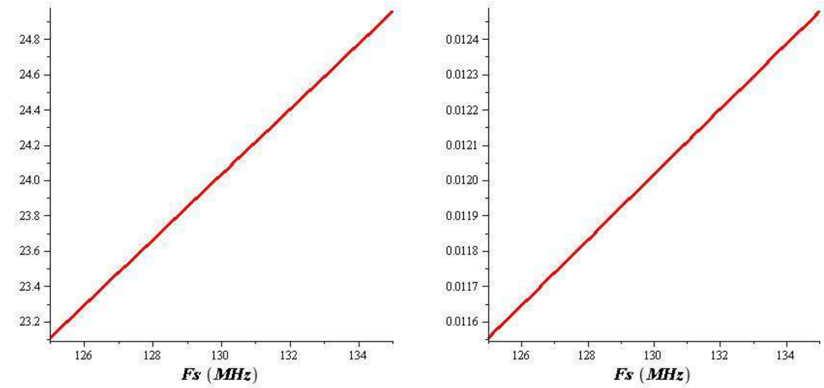


Fig.6 The dependence of ratio of diffractive limit and ultrasonic wavelength ρ_0/λ (dash-blue) and numerical aperture (solid-red) on product $\sqrt{I_S}F_s$.

Finally, we examine the third condition with considering the above investigated microlens is used to focus the flat laser beam with $P_0 = 100mW$ and $P_0 = 0.05mW$. The focused Gaussian beam traps a polystyrene molecule with radius $a = 0.25\mu m$ and $n_p = 1.57$ embedded on water with $n_m = 1.326$. Considering $I_s = 10W/cm^2$, then $\Delta n_0 \approx 5\times10^{-4}$, and from Exp.21, the dependence of $F_{gr,max}$ on chosen ultrasonic frequency, F_s is presented in Fig.7. We can see that in mentioned case $F_{gr,max} = 0.01pN$ when $P_0 = 0.05mW$ only (Fig.7a) and the third condition is satisfied more easily when the laser average power, P_0 is higher (Fig.7b).



a) **b)**

Fig. 7. The dependence of maximum gradient force $F_{gr,max}$ on acoustic frequency F_s with two different average laser power: $P_0 = 100mW$ (a) and $P_0 = 0.05mW$ (b).

From all the obtained results above, the conditions to trap a microparticle of the tweezers using microlens modulated by acoustic waves in EDFG will be satisfied if following principle parameters are chosen:

$$
\begin{aligned}
&EDFG: M = 1.67\times 10^{-14} m^2/W,\ d = 0.25cm \\
&Ultrasonic\ wave: I_s = 10W/cm^2, F_s = (125 \div 135)MHz, V_s = 3500m/s \\
&Laser: \lambda = 1.17\mu m, P_0 = 0.1mW \\
&Bio-molecule: a = 0.25\mu m, n_m = 1.57 \\
&Water: n = 1.326
\end{aligned}
$$

4. CONCLUSIONS AND OUTLOOK

Three necessary conditions for microparticle trapping of optical tweezers using microlens modulated by two acoustic waves in acoustic-elastic medium have been investigated and discussed. For certain case, the trapping capability of optical tweezers is proved and the collection of principle parameters is found out. With this collection of chosen parameters, the microlens 2D array can operate as an optical tweezer 2D array when they are irradiated by plane laser beam.

We can say that all three conditions should be satisfied for the case of the collection of principle parameters found out above. However, the collection can be changed, for example, by the different acoustic wave intensity, consequently, different interval of acoustic wave frequency or different AEM (EDFG can be replaced by GaAs or others) with different thickness. Moreover, in the experiment the water can be replaced by other embedding fluid, which depends on the type of the microparticle. These questions need to be investigated in the future.

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