

# TẠP CHÍ KHOA HỌC ĐẠI HỌC TÂN TRÀO

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# TAP CHÍ KHOA HỌC ĐẠI HỌC TÂN TRÀO LOCAL HOMOLOGY WITH RESPECT TO A PAIR OF IDEALS<br>
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Abstract:

MAD UNIVERSED MANUSCRIPT ON THE FINITENES<br>
Do Ngoc Yen<sup>1,∗</sup><br>
Posts and Telecommunications Institute of Tech<br>
\*Email address: yendn@ptit.edu.vn 1 Posts and Telecommunications Institute of Technology, Ho Chi Minh City, Viet Nam Telecommunications Institute of Technology, Ho Chi Minh City, Viet Nam  $^{*}Email$  and Telecommunications Institute of Technology, Ho Chi Minh A RESULT ON THE FINITENESS OF<br>LOCAL HOMOLOGY WITH RESPE<br>Do Ngoc Yen<sup>1,\*</sup><br><sup>1</sup> Posts and Telecommunications Institute of Technology,<br>\*Email address: yendn@ptit.edu.vn<br>https//doi.org.10.51453/2354-1431/2024/1195 https//doi.org.10.51453/2354-1431/2024/1195 A RESULT ON THE FINITENES<br>
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Firstly, we recall the concept of br veloped strongly for more than 50 years and proved mutative noetherian ring with<br>to be a very important tool in commutative alge-<br>Firstly, we recall the concept of<br>pair content tool in commutative alge-<br>Firstly, we recall to be a very important tool in commutative alge-<br>
bra. There have been many extensions from this modules by<br>
theory. Takahashi, Yoshino and Yoshizawa (2009) donald (Ma<br>
introduced the definition of local cohomology with R % in X Yoshino and Yoshizawa (2009) donald (M<br>
definition of local cohomology with R-module<br>
cof ideals, which is a generalization the zero of logy modules of Grothendieck. Let is a base<br>
i communicative ring, I, J are id % of local cohomology modules of Grothendieck. Let is a base for the<br> *R* is a notherian communicative ring, *I*, *J* are ide-<br>
als of *R* and *M* is an *R*-module. The *i*-th local<br>
in *M* and *M*/*N*<br>
cohomology module

$$
H^i_{I,J}(M) = R^i \Gamma_{I,J}(M) \tag{1}
$$

found the relation between  $H^i_{I,J}(M)$  and  $H^i_I(M)$  = submodules) in M which has the finite intersection<br>local cohomology for a ideal through the isomor-<br>property, then the cosets in  $\mathcal F$  have a non-empty phism  $H_{I,J}^i(M) = R^i \Gamma_{I,J}(M)$  clear base consisting of submod<br>
linearly topologized R-module  $M$ <br>
that  $I^n x \subseteq Jx$  for some integer n. Moreover, they  $\mathcal{F}$  is a family of closed cosets (i.<br>
found the relation between  $H_{I,J}^i(M$ Insearly topologized *R*-module *M*<br>
Insearly topologized *R*-module *M*<br>
Insearly compact if *M* has the follow<br>
that  $I^n x \subseteq Jx$  for some integer *n*. Moreover, they  $\mathcal{F}$  is a family of closed cosets (i.e.<br>
found the

$$
H_{I,J}^i(M) \cong \varinjlim_{\mathfrak{a} \in \widetilde{W}(I,J)} H_{\mathfrak{a}}^i(M) \qquad \text{Next, let } I, J
$$
  
  $R \text{ and } M \text{ an}$ 

where  $\Gamma_{I,J}(M)$  is the set of elements x of M such early compact if M has the follow<br>that  $I^n x \subseteq Jx$  for some integer n. Moreover, they  $\mathcal{F}$  is a family of closed cosets (i.e<br>found the relation between  $H^i_{I,J}(M)$  and  $H_i^{I,J}(M)$ , as follow conomology for a fideal through the isomor-<br>  $H_{I,J}^i(M) \cong \varinjlim_{\mathfrak{a} \in \widetilde{W}(I,J)} H_{\mathfrak{a}}^i(M)$ <br>
iich  $\widetilde{W}(I,J)$  the set of ideals  $\mathfrak{a}$  of  $R$  such that<br>  $\mathfrak{a}+J$  for some integer  $n$  and partial order on<br>  $J$  $H_I, J(M) = \frac{\ln(A \cap H)}{\ln(B \cap H)} H_a(M)$ <br>  $R$  and  $R$ <br>  $(R, J)$  the set of ideals **a** of *R* such that set of ideals or some integer *n* and partial order on integer *s*<br>
etting **a**  $\leq$  **b** if **b**  $\subseteq$  **a** for **a**, **b**  $\in \widetilde{W}(I, J$ *R* and *M* a<br>
a∈w(*I*,*J*)<br>
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a∈w(*I*,*J*)<br>
a∈e and partial order on integer *n* ('<br>  $\leq$  b if  $\mathfrak{b} \subseteq \mathfrak{a}$  for  $\mathfrak{a}, \mathfrak{b} \in \widetilde{W}(I, J)$ . We define<br>
roduce the definition of local<br>
air of ideals  $\label{eq:2.1} \begin{aligned} I^n&\subseteq \mathfrak{a}+J \text{ for some integer $n$ and partial order on} & &\text{integer $n$ (Takahashi, Yoshi;}\\ \widetilde{W}(I,J) \text{ by letting $\mathfrak{a}\leq\mathfrak{b}$ if $\mathfrak{b}\subseteq \mathfrak{a}$ for $\mathfrak{a},\mathfrak{b}\in \widetilde{W}(I,J)$,} &\text{ We define a partial order $\mathfrak{c}$}\\ \text{By duality, we introduce the definition of local} & &\mathfrak{a}\leq\mathfrak{b}$ if $\mathfrak{b}\subseteq \mathfrak{a}$ for $\mathfrak{a},\mathfrak{b}\in \mathrm{$ 

$$
H_i^{I,J}(M) = \varprojlim_{\mathfrak{a} \in \widetilde{W}(I,J)} \mathrm{Tor}_i^R(R/\mathfrak{a},M).
$$

 $\begin{aligned} &\widetilde{W}(I,J) \text{ by letting $\mathfrak a \leq \mathfrak b$ if $\mathfrak b \subseteq \mathfrak a$ for $\mathfrak a$, $\mathfrak b = \mathfrak b$ if $\mathfrak b \subseteq \mathfrak a$ for $\mathfrak a$, $\mathfrak b = \mathfrak b$ if $\mathfrak b \subseteq \mathfrak a$ for $\mathfrak a$, $\mathfrak b = \mathfrak b$ if $\mathfrak b \subseteq \mathfrak a$ for $\mathfrak a$, $\mathfrak b = \mathfrak b$ if $\mathfrak b \subseteq \mathfrak a$ for $\mathfrak a$, $\mathfrak b = \mathfrak b$ if $\mathfrak b \subseteq \mathfrak a$ for $\mathfrak a$ By duality, we introduce the definition of local  $\mathfrak{a} \leq \mathfrak{b}$  if  $\mathfrak{b} \subseteq \mathfrak{a}$  for  $\mathfrak{a}, \mathfrak{b}$ <br>homology for a pair of ideals  $(I, J)$ , denote by we have the homomorphis  $H_i^{I,J}(M)$ , as follow<br> $H_i^{I,J}(M) = \varprojlim_{$  $\begin{array}{lllllllllllllllllll} H_i^{I,J}(M) & \mbox{as follow} & \mbox{for a pair of ideals } (I,J), \mbox{ denote by} & \mbox{we have the homomorphisms.} \label{eq:2.1} & \mbox{Tor}_i^{R_i}(R/\mathfrak{a}^t,M) & \mbox{for all } t>0 \mbox{ are } t\leq T\mbox{ and } t>0 \mbox{ are } t\leq T\mbox{ and } t>0 \mbox{ and } t>0 \mbox{ are } t\leq T\mbox{ and } t>0 \mbox{ and } t>0 \mbox{ are } t\leq T\mbox{ and } t\leq T\mbox{ and } t\$  $\label{eq:1.1} \begin{array}{ll} \mbox{duces a homomorphism of local} \\ H_i^{I,J}(M) = \displaystyle \varprojlim_{\mathfrak{a} \in \widetilde{W}(I,J)} \mathrm{Tor}_i^R(R/\mathfrak{a},M). & H_i^{\mathfrak{b}}(M) \longrightarrow H_i^{\mathfrak{a}}(M). \mbox{ Hence we} \\ \mbox{tem of modules } \{H_i^{\mathfrak{a}}(M)\}_{\mathfrak{a} \in \widetilde{W}(I)} \} & \mbox{dences a homomorphism of local \\ \mbox{models of linearly compact modules. An interest-} \\ \mbox{modules of linearly compact modules. An interest-} \\ \mbox{in module }$  $H_i^{I,J}(M) = \underbrace{\lim}_{\mathfrak{a} \in \widetilde{W}(I,J)} \text{Tor}_i^{\text{rt}}(R/\mathfrak{a},M).$   $H_i^{\mathfrak{b}}(M) \longrightarrow H_i^{\mathfrak{a}}(M).$  Here of modules  $\{H_i^{\mathfrak{a}}(M) \longrightarrow H_i^{\mathfrak{a}}(M)\}$ . Here of modules  $\{H_i^{\mathfrak{a}}(M) \longrightarrow H_i^{\mathfrak{a}}(M)\}$  are modules of linearly compa Essides, we show some basic properties of these following definition.<br>
modules of linearly compact modules. An interest-<br>
ing problem in commutative algebra is the finite-<br>
mess of coassociated primes of local homology.<br> Besides, we show some basic properties of these following definition.<br>
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ness of coassociated primes of local homology.<br>
Yassemi (Yassemi, 1995) defined the set of coas-<br>
sociated prime ideals ing problem in commutative algebra is the finite-<br>ness of coassociated primes of local homology.<br>Tassemi (Yassemi, 1995) defined the set of coas-<br>nodule  $H_i^{I,J}(M)$  of M with res<br>sociated prime ideals of an R-module M, and ness of coassociated primes of local homology.<br>
Yassemi (Yassemi, 1995) defined the set of coas-<br>
sociated prime ideals of an R-module M, and de-<br>
noted by  $\text{Coass}_R(M)$ , to be the set of prime ideals<br>  $\mathbf{p}_i^{I,J}(M) = \lim_{\substack$ Yassemi (Yassemi, 1995) defined the set of coas-<br>
sociated prime ideals of an R-module M, and de-<br>
noted by  $\text{Coass}_R(M)$ , to be the set of prime ideals<br>  $\mathbf{p}$  such that there exists a cocyclic homomorphic<br>
image L of M sociated prime ideals of an R-module M, and denoted by Coass<sub>R</sub>(M), to be the set of prime ideals  $H_i^{I,J}(M) = \lim_{\alpha \in W(I,J)} H_i^{I}$ <br> **P** such that there exists a cocyclic homomorphic image L of M with Ann<sub>R</sub> L = **p**. N. M. Tri noted by Coass<sub>R</sub>(*M*), to be the set of prime ideals<br> **P** such that there exists a cocyclic homomorphic<br>
image *L* of *M* with Ann<sub>R</sub> *L* = **P**. N. M. Tri (Tri,<br>
2021) gave the concept of CFA modules and used it<br> *R*-mo **p** such that there exists a cocyclic homomorphic<br>image L of M with Ann<sub>R</sub> L = **p**. N. M. Tri (Tri, **Proposition 2.2.** Let M be<br>2021) gave the concept of CFA modules and used it  $R$ -module. Then for all  $i \ge$ <br>as a tool to image L of M with Ann<sub>R</sub> L =  $\mathfrak{p}$ . N. M. Tri (Tri, **Proposition 2.2.** Let M be a 2021) gave the concept of CFA modules and used it  $R$ —module. Then for all  $i \geq 0$ , as a tool to study about the fininess of coassocia 2021) gave the concept of CFA modules and<br>as a tool to study about the fininess of coase<br>primes of local homology modules. An R-me<br>is called CFA if there is a submodule N st<br>Cosupp<sub>R</sub>N is a finite set and  $M/N$  is an<br> $R$ -m

introduced the definition of local cohomology with <br> *R*-module. A *nucleus* of *M*<br>
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ule of  $M$  which contain R is a notherian communicative ring,  $I, J$  are ide-<br>
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only if the intersection of all<br>  $(I, J$ 2 MAIN RESULTS<br>2 MAIN RESULTS<br>Throughout this paper, R will always be a com-4\_August 2024| p.33-37<br>2 MAIN RESULTS<br>Throughout this paper, R will always be a com-<br>mutative noetherian ring with non-zero identity.<br>Firstly, we recall the concept of linearly compact 4\_August 2024| p.33-37<br>2 MAIN RESULTS<br>Throughout this paper, R will always be a com-<br>mutative noetherian ring with non-zero identity.<br>Firstly, we recall the concept of linearly compact<br>modules by using the terminology of I 4\_August 2024| p.33-37<br>
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clear base consisting of submodules. A Hausdorff<br>
linearly topologized  $R$ -module  $M$  is said to be *lin-*M is said to be *linearly topologized* if M has a nu-<br>clear base consisting of submodules. A Hausdorff<br>linearly topologized R-module M is said to be *lin-*<br>early compact if M has the following property: if<br> $\mathcal F$  is a fam clear base consisting of submodules. A Hausdorff<br>linearly topologized R-module M is said to be *lin-*<br>early compact if M has the following property: if<br> $\mathcal F$  is a family of closed cosets (i.e. cosets of closed<br>submodules

For  $\widetilde{W}(I,J)$  and  $H^i_I(M)$  and  $H^i_I(M)$  is a pair of ideal through the isomor-<br>
hocal cohomology for a ideal through the isomor-<br>
property, then the cosets in F has<br>
phism<br>  $H^i_{I,J}(M) \cong \lim_{\alpha \in \widetilde{W}(I,J)} H^i_{\mathfrak{a}}(M)$  $\text{Tor}_{i}^{R}(R/\mathfrak{a},M).$   $H_{i}^{\mathfrak{b}}(M) \longrightarrow H_{i}^{\mathfrak{a}}(M).$  Hence we have an inverse sys  $H_i^{(M)}(M) = \lim_{\substack{\alpha \in \widetilde{W}(I,J)}} H_{\alpha}(M)$ <br>
For some integer n and partial order on integer n (Takahashi, Yoshino,  $\widetilde{W}(I,J)$  by letting  $\alpha \leq b$  if  $b \subseteq \alpha$  for  $\alpha, b \in \widetilde{W}(I,J)$ . We define a partial order on  $\widetilde{W}(I,J)$ linearly topologized *R*-module *M* is said to be *lin-*<br>early compact if *M* has the following property: if<br> $\mathcal F$  is a family of closed cosets (i.e. cosets of closed<br>submodules) in *M* which has the finite intersection<br> early compact if M has the following property: if<br>  $\mathcal F$  is a family of closed cosets (i.e. cosets of closed<br>
submodules) in M which has the finite intersection<br>
property, then the cosets in  $\mathcal F$  have a non-empty<br>
inte submodules) in M which has the finite intersection<br>property, then the cosets in F have a non-empty<br>intersection.<br>Next, let I, J be two ideals of the noetherian ring<br> $R$  and M an  $R$ -module. Denote by  $\widetilde{W}(I, J)$  the<br>set submodules) in M which has the finite intersection<br>property, then the cosets in F have a non-empty<br>intersection.<br>Next, let I, J be two ideals of the noetherian ring<br> $R$  and M an  $R$ -module. Denote by  $\widetilde{W}(I, J)$  the<br>set  $_{i}^{R}(R/\mathfrak{b}^{t},M)\longrightarrow$ on-empty<br>
irian ring<br>  $(I, J)$  the<br>
for some<br>
a, 2009).<br>
y letting<br>
f **a** ≤ **b**<br>
, *M*) →<br>
0. It in-<br>
modules<br>
werse sys- $\operatorname{Tor}^R_i(R/\mathfrak{a}^t,M)$  for all  $t > 0$  and i hen the cosets in  $\mathcal F$  have a non-empty<br>
1.<br>
1.  $J$  be two ideals of the noetherian ring<br>
an  $R$ -module. Denote by  $\widetilde{W}(I, J)$  the<br>
s **a** of  $R$  such that  $I^n \subseteq \mathfrak a + J$  for some<br>
Takahashi, Yoshino, Yoshizawa, 2009).<br> intersection.<br>
Next, let I, J be two ideals of the noetherian ring<br>
R and M an R-module. Denote by  $\widetilde{W}(I, J)$  the<br>
set of ideals **a** of R such that  $I^n \subseteq \mathfrak{a}+J$  for some<br>
integer n (Takahashi, Yoshino, Yoshizawa, 20 e two ideals of the noetherian ring<br>  $R$ -module. Denote by  $\widetilde{W}(I, J)$  the<br>
of  $R$  such that  $I^n \subseteq \mathfrak{a}+J$  for some<br>
thashi, Yoshino, Yoshizawa, 2009).<br>
urtial order on  $\widetilde{W}(I, J)$  by letting<br>  $\mathfrak{a}$  for  $\mathfrak{a},$ R and M an R-module. Denote by  $\widetilde{W}(I, J)$ <br>set of ideals **a** of R such that  $I^n \subseteq \mathfrak{a} + J$  for so<br>integer n (Takahashi, Yoshino, Yoshizawa, 200<br>We define a partial order on  $\widetilde{W}(I, J)$  by lett<br> $\mathfrak{a} \leq \mathfrak{b}$  if  $\left\{ \prod_{\mathfrak{a}\in \widetilde{W}(I,J)}\right\}$ . We suggest the by  $\widetilde{W}(I, J)$  the<br>  $\subseteq$   $\mathfrak{a} + J$  for some<br>
oshizawa, 2009).<br>  $(I, J)$  by letting<br>  $I, J$ ). If  $\mathfrak{a} \leq \mathfrak{b}$ <br>  $R_{i}^{R}(R/\mathfrak{b}^{t}, M) \longrightarrow$ <br>
d  $i \geq 0$ . It in-<br>
omology modules<br>
we an inverse sys-<br>
We suggest the<br>
deal set of ideals  $\mathfrak a$  of  $R$  such that  $I^n \subseteq \mathfrak a+J$  for som-<br>integer  $n$  (Takahashi, Yoshino, Yoshizawa, 2009)<br>We define a partial order on  $\widetilde{W}(I,J)$  by letting<br> $\mathfrak a \leq \mathfrak b$  if  $\mathfrak b \subseteq \mathfrak a$  for  $\mathfrak a, \mathfrak b \in \widetilde$ Integer *n* (Takanashi, Toshino, Toshizawa, 2009).<br>
We define a partial order on  $\widetilde{W}(I,J)$  by letting<br>  $\mathfrak{a} \leq \mathfrak{b}$  if  $\mathfrak{b} \subseteq \mathfrak{a}$  for  $\mathfrak{a}, \mathfrak{b} \in \widetilde{W}(I,J)$ . If  $\mathfrak{a} \leq \mathfrak{b}$ <br>
we have the hom We define a partial order on  $W(1, J)$  by letting<br>  $\mathfrak{a} \leq \mathfrak{b}$  if  $\mathfrak{b} \subseteq \mathfrak{a}$  for  $\mathfrak{a}, \mathfrak{b} \in \widetilde{W}(I, J)$ . If  $\mathfrak{a} \leq \mathfrak{b}$ <br>
we have the homomorphisms  $\text{Tor}_i^R(R/\mathfrak{b}^t, M) \rightarrow$ <br>  $\text{Tor}_i^R(R/\mathfrak{a$ be homomorphisms  $\text{Tor}_i^R(R/\mathfrak{b}^t, M) \rightarrow$ <br>the homomorphisms  $\text{Tor}_i^R(R/\mathfrak{b}^t, M) \rightarrow$ <br> $t, M$  for all  $t > 0$  and  $i \geq 0$ . It in-<br>momorphism of local homology modules<br> $\rightarrow H_i^{\mathfrak{a}}(M)$ . Hence we have an inverse sys-<br>dules  $\subseteq$  **u** for **u**, **b**  $\in$  *W* (*t*, *J*). If **u**  $\subseteq$  **b**<br>homomorphisms  $\text{Tor}_i^R(R/\mathfrak{b}^t, M) \longrightarrow$ <br>*A*) for all  $t > 0$  and  $i \ge 0$ . It in-<br>morphism of local homology modules<br> $H_i^{\mathfrak{a}}(M)$ . Hence we have an inverse sys-<br> we have the homomorphisms  $\text{Io}'_i(\Lambda/\mathfrak{b}, M) \rightarrow$ <br>  $\text{Tor}_i^R(R/\mathfrak{a}^t, M)$  for all  $t > 0$  and  $i \geq 0$ . It induces a homomorphism of local homology modules  $H_i^{\mathfrak{b}}(M) \longrightarrow H_i^{\mathfrak{a}}(M)$ . Hence we have an inverse system of

module  $H_i^{I,J}(M)$  of M with respect to (*M*). Hence we have an inverse sys-<br>  ${H_i^{\mathfrak{a}}(M)}_{\mathfrak{a}\in \widetilde{W}(I,J)}$ . We suggest the<br>
ion.<br>
. Let *I*, *J* be two ideals of the ring--module. The *i*-th local homology<br> *I*) of *M* with respect to a pair of<br>
lefined tem of modules  $\{H_i^{\mathfrak{a}}(M)\}_{{\mathfrak{a}}\in \widetilde{W}(I,J)}$ . We suggest the following definition.<br> **Definition 2.1.** Let  $I, J$  be two ideals of the ring  $R$  and  $M$  an  $R$ -module. The  $i$ -th local homology module  $H_i^{I,J}(M)$  of following definition.<br> **Definition 2.1.** Let  $I, J$  be two ideals of the ring  $R$  and  $M$  an  $R$ -module. The  $i$ -th local homology<br>
module  $H_i^{I,J}(M)$  of  $M$  with respect to a pair of<br>
ideals  $(I, J)$  is defined by<br>  $H_i^{I,J}(M)$ wo ideals of the ring<br> *i*—th local homology<br>
respect to a pair of<br>  $H_i^{\mathfrak{a}}(M)$ .<br> *j*)<br> *i* a linearly compact<br>
0,  $H_i^{I,J}(M)$  is a lin-<br>
g & Nam, 2008) that (a) is of the ring<br>
(cal homology<br>
to a pair of<br>
(b).<br>
(cal homology<br>
(*M*) is a lin-<br>
(m)  $(0.2008)$  that **Definition 2.1.** Let *I*, *J* be two ideals of the<br> *R* and *M* an *R*-module. The *i*-th local home<br>
module  $H_i^{I,J}(M)$  of *M* with respect to a pa<br>
ideals  $(I, J)$  is defined by<br>  $H_i^{I,J}(M) = \lim_{a \in \overline{W}(I,J)} H_i^a(M)$ .<br> **Propos** 

$$
H_i^{I,J}(M) = \varprojlim_{\mathfrak{a} \in \widetilde{W}(I,J)} H_i^{\mathfrak{a}}(M).
$$

R-module. Then for all  $i \geqslant 0$ ,  $H_i^{I,J}(M)$  is a lin-

*H* and *M* an *H*-module. Ine *i*-th local nomology<br>
module  $H_i^{I,J}(M)$  of *M* with respect to a pair of<br>
ideals  $(I, J)$  is defined by<br>  $H_i^{I,J}(M) = \lim_{\substack{\alpha \in W(I,J)}} H_i^a(M)$ .<br> **Proposition 2.2.** Let *M* be a linearly compact<br> *R*  ${H_i^{\mathfrak{a}}(M)}_{a \in \widetilde{W}(I,J)}$  forms an inverse or  $M$  with respect to a pair or<br>
and by<br>  $f(x) = \lim_{\alpha \in W(I,J)} H_i^{\alpha}(M).$ <br>
Let  $M$  be a linearly compact<br>
for all  $i \geq 0$ ,  $H_i^{I,J}(M)$  is a lin-<br>
module.<br>
com (Cuong & Nam, 2008) that<br>
forms an inverse system of lin-<br>
dules wit ideals  $(I, J)$  is defined by<br>  $H_i^{I,J}(M) = \lim_{\alpha \in \widetilde{W}(I,J)} H_i^{\alpha}(M)$ .<br> **Proposition 2.2.** Let  $M$  be a linearly compact<br>  $R$ -module. Then for all  $i \geq 0$ ,  $H_i^{I,J}(M)$  is a lin-<br>
early compact  $R$ -module.<br> **Proof.** It follows  $H_i^{I,J}(M) = \underbrace{\lim}_{\mathfrak{a} \in \widetilde{W}(I,J)} H_i^{\mathfrak{a}}(M).$ <br> **Proposition 2.2.** Let *M* be a linearly compa<br> *R*-module. Then for all  $i \ge 0$ ,  $H_i^{I,J}(M)$  is a li<br>
early compact *R*-module.<br> *Proof.* It follows from (Cuong & Nam,  $i^{I,J}(M)$  is also a linearly co =  $\lim_{\alpha \in \widetilde{W}(I,J)} H_i^{\alpha}(M)$ .<br>
Let *M* be a linearly compact<br>
or all  $i \geq 0$ ,  $H_i^{I,J}(M)$  is a lin-<br>
odule.<br>
om (Cuong & Nam, 2008) that<br>
orms an inverse system of lin-<br>
ules with continuos homomor-<br>
(*M*) is also a line **Proposition 2.2.** Let  $M$  be a linearly compact  $R$ –module. Then for all  $i \geq 0$ ,  $H_i^{I,J}(M)$  is a linearly compact  $R$ –module.<br>**Proof.** It follows from (Cuong & Nam, 2008) that  $\{H_i^a(M)\}_{a \in \widetilde{W}(I,J)}$  forms an inverse **Proposition 2.2.** Let  $M$  be a linearly compact  $R$ –module. Then for all  $i \ge 0$ ,  $H_i^{I,J}(M)$  is a linearly compact  $R$ –module.<br> **Proof.** It follows from (Cuong & Nam, 2008) that  $\{H_i^{\mathfrak{a}}(M)\}_{a \in \widetilde{W}(I,J)}$  forms an i *R*–module. Then for all  $i \ge 0$ ,  $H_i^{I,J}(M)$  is a lin-<br>early compact *R*–module.<br><br>*Proof.* It follows from (Cuong & Nam, 2008) that<br> $\{H_i^{\mathfrak{a}}(M)\}_{{a \in \widetilde{W}(I,J)}}$  forms an inverse system of lin-<br>early compact modules

*Do Ngoc Yen/Vol 10.* No 4\_August 2024| p.33-37<br>commuted with inverse limits of inverse systems of and<br>linearly compact R-modules.<br>**Proposition 2.3.** Let  $\{M_t\}$  be an inverse system  $\qquad \qquad \sum_{t=0}^{\lfloor t\rfloor} H_i(\underline{x}(t), \text{Tor}_j$  $Do Ngoc$  Yen/Vol 1<br>commuted with inverse limits of inverse systems<br>linearly compact R−modules.<br>**Proposition 2.3.** Let  $\{M_t\}$  be an inverse syste<br>of linearly compact R−modules with the contin

*Do Ngoc Yen/Vol* 10. No 4\_August 2024| p.33-37<br>
commuted with inverse limits of inverse systems of and<br>
linearly compact  $R$ -modules.<br> **Proposition 2.3.** Let  $\{M_t\}$  be an inverse system of linearly compact  $R$ -modules  $Do Ngoc Yen/Vol 10. No 4_August 2024 | p.33-37$ <br>commuted with inverse limits of inverse systems of and<br>linearly compact R−modules.<br>**Proposition 2.3.** Let  $\{M_t\}$  be an inverse system of linearly compact R−modules with the continu-<br>ous *Do Ngoc Yen/*Vol 10. No 4\_Augu<br>
commuted with inverse limits of inverse systems of and<br>
linearly compact  $R$ -modules.<br> **Proposition 2.3.** Let  $\{M_t\}$  be an inverse system of linearly compact  $R$ -modules with the continu commuted with inverse limits of inverse systems of and<br>
linearly compact  $R$ -modules.<br> **Proposition 2.3.** Let  $\{M_t\}$  be an inverse system<br>
of linearly compact  $R$ -modules with the continu-<br>
ous homomorphisms. Then<br>
when

$$
H_i^{I,J}(\varprojlim_t M_t) \cong \varprojlim_t H_i^{I,J}(M_t).
$$
 for all  $\mathfrak{b}$   
  $\varprojlim$ 

**Proposition 2.3.** Let 
$$
\{M_t\}
$$
 be an inverse system  
\nof linearly compact  $R$ -modules with the continu-  
\nous homomorphisms. Then  
\n $H_i^{I,J}(\underbrace{\lim}_{t} M_t) \cong \underbrace{\lim}_{t} H_i^{I,J}(M_t).$   
\n**Proof.** Note that inverse limits are commuted and  
\n $H_i^{I,J}(\underbrace{\lim}_{t} M_t) = \underbrace{\lim}_{\substack{\alpha \in \widetilde{W}(I,J) \\ \beta \in \widetilde{W}(I,J)}} H_i^{q}(M_t).$   
\nby (Cuang & Nam, 2008) we have  
\n $H_i^{I,J}(\underbrace{\lim}_{t} M_t) = \underbrace{\lim}_{\substack{\alpha \in \widetilde{W}(I,J) \\ \alpha \in \widetilde{W}(I,J)}} H_i^{q}(\underbrace{\lim}_{t} M_t)$   
\n $\cong \underbrace{\lim}_{\substack{\alpha \in \widetilde{W}(I,J) \\ \beta \in \widetilde{W}(I,J)}} H_i^{q}(\underbrace{\lim}_{t} M_t)$   
\n $\cong \underbrace{\lim}_{\substack{\alpha \in \widetilde{W}(I,J) \\ \beta \in \widetilde{W}(I,J)}} H_i^{q}(M_t)$   
\n $\cong \underbrace{\lim}_{\substack{\alpha \in \widetilde{W}(I,J) \\ \beta \in \widetilde{W}(I,J)}} H_i^{q}(M_t)$   
\n $\cong \underbrace{\lim}_{t} H_i^{I,J}(M_t).$   
\n**Lemma 2.4.** Let  $M$  be an linearly compact for all  $t > 0$ , because  $\{a^tM\} \in \widetilde{W}(I, J)$   
\n $H_i^{I,J}(H_j^{I,J}(M)) \cong \begin{cases} H_j^{I,J}(M), & i = 0 \\ 0, & i > 0 \end{cases}$  we have a short exact sequence  
\n $H_i^{I,J}(H_j^{I,J}(M)) \cong \begin{cases} H_j^{I,J}(M), & i = 0 \\ 0, & i > 0 \end{cases}$  we have a short exact sequence

 $R$ -module. Then for all  $j \ge 0$ , early compact modules, by (Cuong & Nam, 2008)

$$
H_i^{I,J}(H_j^{I,J}(M)) \cong \begin{cases} H_j^{I,J}(M), & i = 0 \\ 0, & i > 0 \end{cases}
$$

**Example 1.**  $H_i^u(M_t)$ <br>  $=\varprojlim_t H_i^{I,J}(M_t).$ <br> **Lemma 2.4.** Let *M* be an linearly compact for all  $t > 0$ , because<br> *R*-module. Then for all  $j \ge 0$ ,<br>  $H_i^{I,J}(H_j^{I,J}(M)) \cong \begin{cases} H_j^{I,J}(M), & i = 0 \\ 0, & i > 0 \end{cases}$  we have a short example  $j^{\mathfrak{a}}(M)$  is Lemma 2.4. Let M be an linearly compact for all  $t > 0$ , because  $\{a^tM\}$  is in <br>  $R$ -module. Then for all  $j \ge 0$ ,<br>  $H_i^{I,J}(H_j^{I,J}(M)) \cong \begin{cases} H_j^{I,J}(M), & i = 0 \\ 0, & i > 0 \end{cases}$  we have a short exact sequence of  $H_i^{I,J}(H_j^{I,J}(M)) \con$ **Lemma 2.4.** Let *M* be an linearly compact for all <br> *R*-module. Then for all  $j \ge 0$ , early compact for all  $R$ -module. Then for all  $j \ge 0$ , early compact for all  $j \ge 0$ , early compact  $H_i^{I,J}(H_j^{I,J}(M)) \cong \begin{cases} H_j^{I,J}(M), &$ 

**Lemma 2.4.** Let *M* be an linearly compact for all 
$$
t > 0
$$
, because  $\{a$   
\n
$$
H_i^{I,J}(H_j^{I,J}(M)) \cong \begin{cases} H_j^{I,J}(M), & i = 0 \\ 0, & i > 0 \end{cases}
$$
 we have a short exact s  
\n*Proof.* By the (Cuang & Nam, 2008),  $H_j^{\alpha}(M)$  is  
\nlinearly compact for all  $\mathfrak{a} \in \widetilde{W}(I, J)$ . Then we have  
\nby (Cuang & Nam, 2008)  
\n
$$
H_i^{I,J}(H_j^{I,J}(M)) = \lim_{\mathfrak{b} \in \widetilde{W}(I, J)} H_j^{\alpha}(M)
$$
 for  $\mathfrak{b} \in \widetilde{W}(I, J)$  for  $\mathfrak{b} \in \widetilde{W}(I, J)$   
\n
$$
\cong \lim_{\mathfrak{b} \in \widetilde{W}(I, J)} \lim_{\mathfrak{a} \in \widetilde{W}(I, J)} H_i^{\alpha}(H_j^{\alpha}(M))
$$
 Hence we get a long ex-  
\n
$$
\cong \lim_{\mathfrak{b} \in \widetilde{W}(I, J)} \lim_{\mathfrak{a} \in \widetilde{W}(I, J)} H_i^{\alpha}(H_j^{\alpha}(M))
$$
 Hence we get a long ex-  
\n
$$
\cong \lim_{\mathfrak{a} \in \widetilde{W}(I, J)} \lim_{\mathfrak{a} \in \widetilde{W}(I, J)} H_i^{\alpha}(H_j^{\alpha}(M))
$$
  $\cdots \to H_{i+1}^{I,J}(\Lambda_{I,J}(M))$   
\nFrom 2.7 (Cuong & Nam, 2008), we have  
\n
$$
H_i^{\alpha}(H_j^{\alpha}(M)) = \lim_{\substack{\mathfrak{a} \in \widetilde{W}(I, J)} \operatorname{Tor}_i^R(R/\mathfrak{b}^t, \lim_{\substack{\mathfrak{b} \in \widetilde{W}(I, J)} \operatorname{Tor}_j^R(R/R(\mathfrak{a}^s, M))} \to H_1(\Lambda_{I,J}(M)) \to
$$
  
\n
$$
\cong \lim_{\substack{\mathfrak{b} \in \widetilde{V}(I, J
$$

H<sup>b</sup> i (H a j i (R/b t , lim ←−<sup>s</sup> j (R/a<sup>s</sup> lim←−<sup>s</sup> Tor<sup>R</sup> i (R/b t , Tor<sup>R</sup> j (R/a<sup>s</sup> lim←−<sup>t</sup> Tor<sup>R</sup> i (R/b t , Tor<sup>R</sup> j (R/a<sup>s</sup> Let <sup>x</sup> = (<sup>x</sup>1, . . . , xr) is a system of generator of <sup>b</sup> and <sup>x</sup>(t) = (<sup>x</sup> , . . . , xtr Nam, 2008) that Then for all <sup>b</sup> <sup>a</sup>, we have <sup>x</sup>(t) <sup>⊆</sup> <sup>b</sup>

and  $x(t) = (x_1^t, \ldots, x_r^t)$ . It follows from (Cuong &

$$
H_i^{\mathfrak{b}}(H_j^{\mathfrak{a}}(M)) \cong \varprojlim_s \varprojlim_t H_i(\underline{x}(t), \operatorname{Tor}_j^R(R/\mathfrak{a}^s, M)) \qquad \textbf{Theorem 2.6.}
$$

Then for all  $\mathfrak{h} \geq a$ , we have  $\underline{x}(t) \subseteq \mathfrak{b}^t \subseteq \mathfrak{a}^t \subseteq \mathfrak{a}^s$ , for  $H_i^{r,\infty}(M)$  are CFA for all  $i$ <br>all  $t > s$ . Hence  $x(t) \text{Tor}^R(R/\mathfrak{a}^s | M) = 0 \forall t > s$ , ideal  $\mathfrak{a} \in \widetilde{W}(I, J)$  such that all  $t \geqslant s$ . Hence  $\underline{x}(t) \text{Tor}_j^R(R/\mathfrak{a}^s, M) = 0, \forall t \geqslant s$ . ideal  $\begin{aligned}\n&\equiv \varprojlim_{s} \varprojlim_{t} \text{Tor}_{i}^{r}(R/\mathfrak{b}^{r}, \text{Tor}_{j}^{r}(R/\mathfrak{a}^{r}, M))\n\end{aligned}$ The lemma now follows<br>
Let  $\underline{x} = (x_{1}, \ldots, x_{r})$  is a system of generator of  $\mathfrak{b}$  The following result gi<br>
and  $\underline{x}(t) = (x_{1}^{t}, \ldots, x_{r}^{t})$ It follows from (Cuong & finiteness of coass<br>
with respect to a<br>  $H_i(\underline{x}(t), \text{Tor}_j^R(R/\mathfrak{a}^s, M))$ <br>  $\begin{array}{ll}\n\text{Theorem 2.6.} \\
\text{Theorem 2.6.} \\
\text{R-modele and } t \\
\text{We } \underline{x}(t) \subseteq \mathfrak{b}^t \subseteq \mathfrak{a}^t \subseteq \mathfrak{a}^s, \text{ for } H_i^{I,J}(M) \text{ are } \text{CF}_i^R(R/\mathfrak{a}^$ 

$$
\varprojlim_{t} H_0(\underline{x}(t), \operatorname{Tor}_j^R(R/\mathfrak{a}^s, M)) \cong \operatorname{Tor}_j^R(R/\mathfrak{a}^s, M)
$$

and

$$
\varprojlim_t H_i(\underline{x}(t), \operatorname{Tor}_j^R(R/\mathfrak{a}^s, M)) = 0, \forall i > 0.
$$

 $I,J(M_t)$ , for all  $\mathfrak{b} \geqslant \mathfrak{a}$ . Therefore,  $H_0^{I,J}(H_j^{I,J}(M)) \cong$ commuted with inverse limits of inverse systems of<br>
linearly compact  $R$ -modules.<br> **Proposition 2.3.** Let  $\{M_t\}$  be an inverse system<br>
of linearly compact  $R$ -modules with the continu-<br>
ous homomorphisms. Then<br>  $H_i^{I,J}(H$ 4\_August 2024| p.33-37<br>
and<br>  $\lim_{t \to t} H_i(\underline{x}(t), \text{Tor}_j^R(R/\mathfrak{a}^s, M)) = 0, \forall i > 0.$ So we have  $H_i^{I,J}(H_j^{I,J}(M)) = 0$  for all  $i > 0$ .<br>
When  $i = 0$ , we proved  $H_0^{\mathfrak{h}}(H_j^{\mathfrak{a}}(M)) \cong H_j^{\mathfrak{a}}(M)$ <br>
for all  $\mathfrak{b} \geq \mathfrak{a$  $I_i^{I,J}(H_j^{I,J}(M)) = 0$  for all  $i > 0$ . 4\_August 2024| p.33-37<br>
and<br>  $\varprojlim_t H_i(\underline{x}(t), \text{Tor}_j^R(R/\mathfrak{a}^s, M)) = 0, \forall i > 0.$ So we have  $H_i^{I,J}(H_j^{I,J}(M)) = 0$  for all  $i > 0$ .<br>
When  $i = 0$ , we proved  $H_0^{\mathfrak{h}}(H_j^{\mathfrak{a}}(M)) \cong H_j^{\mathfrak{a}}(M)$ <br>
for all  $\mathfrak{b} \geq \mathfrak{a}$ .  $\Gamma_0^{\mathfrak{b}}(H_j^{\mathfrak{a}}(M)) \cong H_j^{\mathfrak{a}}(M)$ 4\_August 2024| p.33-37<br>
and<br>  $\varprojlim_t H_i(\underline{x}(t), \text{Tor}_j^R(R/\mathfrak{a}^s, M)) = 0, \forall i > 0$ So we have  $H_i^{I,J}(H_j^{I,J}(M)) = 0$  for all  $i$ <br>
When  $i = 0$ , we proved  $H_0^{\mathfrak{b}}(H_j^{\mathfrak{a}}(M)) \cong L$ <br>
for all  $\mathfrak{b} \geqslant \mathfrak{a}$ . Therefore,  $H_$  $\lim_{\substack{\longrightarrow \\ \widetilde{\mathcal{M}}(I,J)\in\widetilde{\mathcal{M}}(I,J)}} H_j^{\mathfrak{a}}(M) = H_j^{I,J}(M).$ and<br>  $\lim_{t \to 0} H_i(\underline{x}(t), \text{Tor}_j^R(R/\mathfrak{a}^s, M)) = 0, \forall i > 0.$ <br>
So we have  $H_i^{I,J}(H_j^{I,J}(M)) = 0$  for all  $i$ <br>
When  $i = 0$ , we proved  $H_0^{\mathfrak{b}}(H_j^{\mathfrak{a}}(M)) \cong H_j^{\mathfrak{a}}$ <br>
for all  $\mathfrak{b} \geqslant \mathfrak{a}$ . Therefore,  $H_0^{I,J}(H_j^{I$  $\mathfrak{a}\in \stackrel{\sim}{W}(I,J)$  b∈ $\stackrel{\sim}{W}(I,J)$ 3-37<br>  $\binom{R}{j}(R/\mathfrak{a}^s, M)) = 0, \forall i > 0.$ <br>  $\binom{I, J}{j}(M) = 0$  for all  $i > 0.$ <br>
coved  $H_0^{\mathfrak{b}}(H_j^{\mathfrak{a}}(M)) \cong H_j^{\mathfrak{a}}(M)$ <br>
herefore,  $H_0^{I, J}(H_j^{I, J}(M)) \cong$ <br>  $(M) = H_j^{I, J}(M)$ .<br>  $M$  be an linearly compact and<br>  $\lim_{t \to i} H_i(\underline{x}(t), \text{Tor}_j^R(R/\mathfrak{a}^s, M)) = 0, \forall i > 0.$ <br>
So we have  $H_i^{I,J}(H_j^{I,J}(M)) = 0$  for all  $i > 0$ .<br>
When  $i = 0$ , we proved  $H_0^b(H_j^{\mathfrak{a}}(M)) \cong H_j^{\mathfrak{a}}(M)$ <br>
for all  $\mathfrak{b} \geq \mathfrak{a}$ . Therefore,  $H_0^{I,J}(H_j^{I,J}(M$  $\begin{aligned} &\varprojlim_{t}H_{i}(\underline{x}(t),\text{Tor}_{j}^{R}(R/\mathfrak{a}^{s},M))=0,\forall i>0.\\ \text{So we have } H_{i}^{I,J}(H_{j}^{I,J}(M))=0\text{ for all }i>0.\\ \text{When }i=0,\text{ we proved }H_{0}^{\mathfrak{b}}(H_{j}^{\mathfrak{a}}(M))\cong H_{j}^{\mathfrak{a}}(M)\\ \text{for all }\mathfrak{b}\ \geqslant\ \mathfrak{a}. \text{ Therefore, }H_{0}^{I,J}(H_{j}^{I,J}(M))\cong\\ &\varprojlim_{\mathfrak{a}\$  $(0, \int_0^b (H_j^{\mathfrak{a}}(M)) \cong H_j^{\mathfrak{a}}(M)$ <br>  $(0, \int_0^b H_j^{I,J}(H_j^{I,J}(M)) \cong H_j^{I,J}(M).$ <br>
an linearly compact<br>  $(0, \int_0^b H_i^{I,J}(M), i > 0)$ <br>  $(0, \int_0^b H_i^{I,J}(M), i > 0)$ 0 for all  $i > 0$ .<br>  $\binom{n}{M} \cong H_j^a(M)$ <br>  $\binom{I,J}{0}(H_j^{I,J}(M)) \cong M$ .<br>  $\binom{n}{M}$ .<br>
linearly compact<br>  $i = 0$ <br>  $(M), i > 0$ .<br>
have short exact for all  $\mathfrak{b} \geq \mathfrak{a}$ . Therefore,  $H_0^{I,J}(H_j^{I,J}(M)) \cong \lim_{\mathfrak{a} \in \widetilde{W}(I,J)} \lim_{\mathfrak{b} \in \widetilde{W}(I,J)} H_j^{\mathfrak{a}}(M) = H_j^{I,J}(M).$ <br> **Lemma 2.5.** Let  $M$  be an linearly compact  $R$ -module. Then<br>  $H_i^{I,J}(\bigcap_{\mathfrak{a} \in \widetilde{W}(I,J)} \$ 

$$
\lim_{\substack{\mathfrak{a}\in W(I,J)\text{ is commuted and}\\{\left(\varprojlim M_t\right)}}} \frac{\varprojlim}{\mathfrak{a}\in \widetilde{W}(I,J)} H_j^{\mathfrak{a}}(M) = H_j^{I,J}(M).
$$
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$$
\text{Lemma 2.5. Let } M \text{ be an linearly compact}
$$
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$$
\lim_{\substack{\mathfrak{a}\in W(I,J)\text{ is a \text{ odd}}} H_i^{I,J}(\bigcap_{\mathfrak{a}\in \widetilde{W}(I,J)} \mathfrak{a} M) \cong \left\{ \begin{array}{ll} 0, & i=0\\ H_i^{I,J}(M), & i>0 \end{array} \right. \cdot
$$
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$$
H_i^{\mathfrak{a}}(M_t) \qquad \text{Proof. For each } \mathfrak{a} \in \widetilde{W}(I,J), \text{ we have short exact}
$$
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\text{sequence of linearly compact } R-\text{module}
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t).
$$
\n
$$
0 \to \mathfrak{a}^t M \to M \to M/\mathfrak{a}^t M \to 0,
$$

sequence of linearly compact R-module

$$
0 \to \mathfrak{a}^t M \to M \to M/\mathfrak{a}^t M \to 0,
$$

 $\frac{\text{im}(H_i^{\mathfrak{a}}(M_t))}{t}$   $\frac{\text{im}(H_i^{\mathfrak{a}}(M_t))}{t}$   $\frac{\text{Proof. For each } \mathfrak{a} \in \widetilde{W}(I,J)}{\text{sequence of linearly compact}}$ <br>  $M_t$ .<br>  $M_t$ .<br>  $0 \to \mathfrak{a}^t M \to M \to M$ <br>  $\frac{\text{linearly compact}}{\text{empty compact}}$  for all  $t > 0$ , because  $\{\mathfrak{a}^t M\}$  is<br>  $\frac{\text{early compact modules, by (we have a short exact sequence}}{\text{modules}}$ <br> (*I,J*)  $\iint_{\mathbb{R}^n} H_i^{\mathfrak{a}}(M_t)$  **Proof.** For each  $\mathfrak{a} \in \widetilde{W}(I, J)$ <br>
sequence of linearly compare  $H_i^{I,J}(M_t)$ .  $0 \to \mathfrak{a}^t M \to M -$ <br>
i an linearly compact for all  $t > 0$ , because  $\{\mathfrak{a}^t M\}$ <br>  $\geq 0$ , early linearly compact for all  $t > 0$ , because {<br>
early compact modul<br>
(*M*),  $i = 0$  we have a short exact<br>  $i > 0$  modules<br>
., 2008),  $H_j^{\mathfrak{a}}(M)$  is<br>  $\left(\begin{array}{cc} 2008 \\ j \end{array}\right)$ ,  $H_j^{\mathfrak{a}}(M)$  is<br>
From that, we have s<br>
( $\$  $R$ -module. Then<br>  $H_i^{I,J}(\bigcap_{\mathfrak{a}\in \widetilde{W}(I,J)}\mathfrak{a} M)\cong \left\{ \begin{array}{ll} 0, & i=0\\ H_i^{I,J}(M), & i>0 \end{array} \right.$ <br> **Proof.** For each  $\mathfrak{a}\in \widetilde{W}(I,J)$ , we have short exact<br>
sequence of linearly compact  $R$ -module<br>  $0 \to \mathfrak{a}^t M \to M \to$  $\cong \left\{ \begin{array}{ll} 0, & i=0 \ H_{i}^{I,J}(M), & i>0 \end{array} \right.$ <br>  $\tilde{M}(I,J),$  we have short exact<br>
mpact  $R$ -module<br>  $M \to M/\mathfrak{a}^{t}M \to 0,$ <br>  $M^{t}M$  is inverse system of lin-<br>  $k$ , by (Cuong & Nam, 2008)<br>
equence of linearly compact  $H_i^{I,J}(\bigcap_{\mathfrak{a}\in \widetilde{W}(I,J)}\mathfrak{a}\,M)\cong \left\{\begin{array}{ll} 0, & i=0\\ H_i^{I,J}(M), & i>0 \end{array}\right..$ <br> **Proof.** For each  $\mathfrak{a}\in \widetilde{W}(I,J)$ , we have short exact sequence of linearly compact  $R$ —module<br>  $0\to \mathfrak{a}^tM\to M\to M/\mathfrak{a}^tM\to 0,$  fo  $H_i^{I,J}(\bigcap_{\mathfrak{a}\in\widetilde{W}(I,J)}\mathfrak{a} M)\cong\begin{cases} 0, & i=0 \ H_i^{I,J}(M), & i>0 \end{cases}.$ <br> **Proof.** For each  $\mathfrak{a}\in\widetilde{W}(I,J)$ , we have short exact sequence of linearly compact  $R$ —module<br>  $0 \to \mathfrak{a}^t M \to M \to M/\mathfrak{a}^t M \to 0,$ <br>
for all modules  $0 \to \mathfrak{a}^t M \to M \to M/\mathfrak{a}^t M \to 0,$ <br>for all  $t > 0$ , because  $\{\mathfrak{a}^t M\}$  is inverse system of lin-<br>early compact modules, by (Cuong & Nam, 2008)<br>we have a short exact sequence of linearly compact<br>modules<br> $0 \to \bigcap_{t>0} \$ pact modules, by (Cuong & Nam, 2008)<br>
short exact sequence of linearly compact<br>
→  $\bigcap_{t>0}$   $\mathfrak{a}^t M \to M \to \Lambda_{\mathfrak{a}}(M) \to 0$ .<br>
t, we have sequence<br>  $\bigcap_{\mathfrak{b} \in \widetilde{W}(I,J)} \mathfrak{b} M \to M \to \Lambda_{I,J}(M) \to 0$ .<br>
get a long exact b M →  $M \rightarrow M \rightarrow \Lambda_{\mathfrak{a}}(M) \rightarrow 0$ .<br>
From that, we have sequence<br>  $0 \rightarrow \bigcap_{t>0} \mathfrak{a}^t M \rightarrow M \rightarrow \Lambda_{\mathfrak{a}}(M) \rightarrow 0$ .<br>
From that, we have sequence<br>  $0 \rightarrow \bigcap_{\mathfrak{b} \in \widetilde{W}(I,J)} \mathfrak{b} M \rightarrow M \rightarrow \Lambda_{I,J}(M) \rightarrow 0$ .<br>
Hence we get a long exact

$$
0 \to \bigcap_{t>0} \mathfrak{a}^t M \to M \to \Lambda_{\mathfrak{a}}(M) \to 0.
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H_j^{\mathfrak{a}}(M)
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H_j^{\mathfrak{a}}(M)
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H_j^{\mathfrak{a}}(M)
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H_j^{\mathfrak{a}}(M)
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H_j^{\mathfrak{a}}(M)
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H_j^{\mathfrak{a}}(M)
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H_j^{\mathfrak{a}}(M)
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 (9)  
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$$
\cdots \rightarrow H_{i+1}^{I,J}(\Lambda_{I,J}(M)) \rightarrow H_i^{I,J}(\bigcap_{h \in \widetilde{W}(I, I)} \mathfrak{b}(M) \rightarrow
$$

 $\lim_{\substack{\longleftarrow \\ \overline{\omega}(I,I)}} H_i^{\mathfrak{b}}(H_j^{\mathfrak{a}}(M))$  Hence we get a long

$$
M)) = \lim_{b \in \overline{W}(I,J)} H_{i}^{b}(\lim_{\alpha \in \overline{W}(I,J)} H_{i}^{a}(M)) \longrightarrow 0 \longrightarrow \bigcap_{b \in \overline{W}(I,J)} bM \rightarrow M \rightarrow \Lambda_{I,J}(M) \rightarrow 0.
$$
  
\n
$$
\cong \lim_{b \in \overline{W}(I,J)} \lim_{\alpha \in \overline{W}(I,J)} H_{i}^{b}(H_{j}^{a}(M)) \longrightarrow H_{i+1}^{I,J}(\Lambda_{I,J}(M)) \rightarrow H_{i}^{I,J}(\bigcap_{b \in \overline{W}(I,J)} bM) \rightarrow
$$
  
\n
$$
\cong \lim_{\alpha \in \overline{W}(I,J)} \lim_{b \in \overline{W}(I,J)} H_{i}^{b}(H_{j}^{a}(M)) \longrightarrow H_{i+1}^{I,J}(\Lambda_{I,J}(M)) \rightarrow H_{i}^{I,J}(\bigcap_{b \in \overline{W}(I,J)} bM) \rightarrow
$$
  
\n
$$
\cong \lim_{\alpha \in \overline{W}(I,J)} \text{Tor}_{i}^{R}(R/b^{t}, \lim_{\alpha \in \overline{W}(I)} (\text{Tor}_{j}^{R}(R/\mathfrak{a}^{s}, M))) \longrightarrow H_{1}(\Lambda_{I,J}(M)) \rightarrow H_{0}^{I,J}(\bigcap_{b \in \overline{W}(I,J)} bM) \rightarrow
$$
  
\n
$$
\cong \lim_{\overline{t} \to \overline{t}} \text{Tor}_{i}^{R}(R/b^{t}, \lim_{\alpha \in \overline{X}} (R/\mathfrak{a}^{s}, M)) \longrightarrow H_{0}^{I,J}(M) \rightarrow H_{0}^{I,J}(\Lambda_{I,J}(M)) \rightarrow 0.
$$
  
\n
$$
\cong \lim_{\overline{t} \to \overline{t}} \text{Tor}_{i}^{R}(R/b^{t}, \text{Tor}_{j}^{R}(R/\mathfrak{a}^{s}, M)) \longrightarrow H_{0}^{I,J}(M) \rightarrow H_{0}^{I,J}(\Lambda_{I,J}(M)) \rightarrow 0.
$$
  
\n
$$
\cong \lim_{\alpha \in \overline{t} \to \overline{t}} \text{Im } \text{Tor}_{i}^{R}(R/b^{t}, \text{Tor}_{j}^{R}(R/\mathfrak{a}^{s}, M)) \longrightarrow H_{0}^{I,J}(M) \rightarrow H_{0}^{I,J}(\Lambda_{I,J}(
$$

 $\rightarrow H_1(\Lambda_{I,J}(M)) \rightarrow H_0$  (<br>  $\star^s, M$ ))  $\rightarrow H_0^{I,J}(M) \rightarrow H_0^{I,J}(\Lambda_I$ <br>  $\star^s, M$ )) The lemma now follows from L<br>
or of b The following result gives us<br>
finiteness of coassociated prime<br>
with respect to a pair of ideals<br>  $(M)$  Theore  $t \subseteq \mathfrak{a}^s$ , for  $H_i^{I,J}(M)$  are CFA for all  $i < t$ , (*M*)  $\rightarrow H_0^{I,J}(M) \rightarrow H_0^{I,J}(\Lambda_{I,J}(M))$ <br>
The lemma now follows from Lem<br>
of b The following result gives us a c<br>
finiteness of coassociated primes o<br>
with respect to a pair of ideals.<br>
(*I*) **Theorem 2.6.** Let *M* be a li<br> t, Tor<sub>j</sub><sup>t</sup>(R/ $\mathfrak{a}^s$ , M)) The lemma now follows from<br>
of generator of  $\mathfrak{b}$  The following result gives<br>
vs from (Cuong & finiteness of coassociated p<br>
with respect to a pair of id<br>
Tor<sub>j</sub><sup>R</sup>(R/ $\mathfrak{a}^s$ , M)) ong & finiteness of coassociated primes<br>with respect to a pair of ideals.<br>M) <br>**Theorem 2.6.** Let M be a<br> $R$ -module and t a non-negative<br> $\mathfrak{a}^s$ , for  $H_i^{I,J}(M)$  are CFA for all  $i <$ <br> $t \geqslant s$ . ideal  $\mathfrak{a} \in \widetilde{W}(I,J$  $\rightarrow H_1(M_1, J(M)) \rightarrow H_0$  ( | | 0 $M$ )  $\rightarrow$ <br>  $\downarrow \downarrow \downarrow \downarrow \downarrow$   $\downarrow \downarrow \downarrow \downarrow$   $\rightarrow H_0^{I,J}(M) \rightarrow H_0^{I,J}(\Lambda_{I,J}(M)) \rightarrow 0$ .<br>
The lemma now follows from Lemma 2.4.<br>
The following result gives us a condition for the<br>
finiteness of coassoci  $\rightarrow H_0^{I,J}(M) \rightarrow H_0^{I,J}(\Lambda_{I,J}(M)) \rightarrow 0.$ <br>lemma now follows from Lemma 2.4.<br>following result gives us a condition for the<br>mess of coassociated primes of local homology<br>respect to a pair of ideals.<br>**orem 2.6.** Let  $M$  be a linea  $\rightarrow H_0^{(1)}(M) \rightarrow H_0^{(1)}(\Lambda_{I,J}(M)) \rightarrow 0.$ <br>The lemma now follows from Lemma 2.4.<br>The following result gives us a condition for the<br>finiteness of coassociated primes of local homology<br>with respect to a pair of ideals.<br>**Theorem 2** finiteness of coassociated primes of local homology<br>with respect to a pair of ideals.<br>**Theorem 2.6.** Let  $M$  be a linearly compact<br> $R$ -module and  $t$  a non-negative integer. If  $M$  and<br> $H_i^{I,J}(M)$  are CFA for all  $i < t$ , th (b) is finite.<br>
(M)) is finite.

$$
R/\mathfrak{a}\mathop{{\otimes}_R}H_t^{I,J}(M)
$$

 $j(R/\mathfrak{a}^s, M)$  is CFA. In particular,  $\text{Coass}_R(H_t^{I,J}(M))$  is finite.

*Do Ngoc Yen/*Vol 10. No 4\_August 2024| p.33-37<br> **Proof.** The proof is by induction on t. Let  $t = 0$ . of these modules and gave a result<br>
There is an ideal  $\mathfrak{a} \in \tilde{W}(I, J)$  such that ∩  $\mathfrak{a} M = \begin{array}{c}$  of coassocia *Do Ngoc Yen/Vol* 10. No 4\_Augu<br> **Proof.** The proof is by induction on t. Let  $t = 0$ . of these<br>
There is an ideal  $\mathfrak{a} \in \tilde{W}(I, J)$  such that  $\cap \mathfrak{a} M =$  of coasse<br>  $\mathfrak{a} M$ . The short exact sequence<br>  $0 \to \mathfrak{a} M$ *Do Ngoc Yen/*Vol 10. No 4\_August 20.<br> **Proof.** The proof is by induction on t. Let  $t = 0$ . of these modu<br>
There is an ideal  $\mathfrak{a} \in \tilde{W}(I, J)$  such that  $\cap \mathfrak{a} M =$  of coassociate<br>  $\mathfrak{a} M$ . The short exact sequen

$$
0 \to \mathfrak{a} \, M \to M \to H_0^{1,J}(M) \to 0
$$

$$
R/\mathfrak{a}\otimes_R \mathfrak{a} M \to R/\mathfrak{a}\otimes_R M \to R/\mathfrak{a}\otimes_R H_0^{I,J}(M) \to 0.
$$

**Proof.** The proof is by induction on t. Let  $t = 0$ . of these modules and gave a result<br>
There is an ideal  $\mathfrak{a} \in \tilde{W}(I, J)$  such that  $\cap \mathfrak{a} M$  = of coassociated primes of these modules and<br>  $\mathfrak{a} M$ . The short  $_{0}^{I,J}(M)$  is CFA. by induction on *t*. Let  $t = 0$ . of these module<br>  $\tilde{W}(I, J)$  such that  $\cap \mathfrak{a} M =$  of coassociated<br>
sequence<br>  $M \rightarrow H_0^{I, J}(M) \rightarrow 0$  funded by the H<br>
tude of Techno<br>
exact sequence<br>  $\otimes_R M \rightarrow R/\mathfrak{a} \otimes_R H_0^{I, J}(M) \rightarrow 0$ .<br> There is an ideal  $\mathfrak{a} \in W(I, J)$  such that  $\cap \mathfrak{a} M =$ <br>  $\mathfrak{a} M$ . The short exact sequence<br>  $0 \to \mathfrak{a} M \to M \to H_0^{I, J}(M) \to 0$ <br>
induces the following exact sequence<br>  $R/\mathfrak{a} \otimes_R \mathfrak{a} M \to R/\mathfrak{a} \otimes_R M \to R/\mathfrak{a} \otimes_R H$  $H_t^{I,J}(M) \cong H_t^{I,J}(\cap \mathfrak{a} M) =$  Cuong N. 1  $H_t^{I,J}(\mathfrak{a} M)$ , we can replace M by The short exact sequence<br>  $0 \rightarrow a \, M \rightarrow M \rightarrow H_0^{I,J}(M) \rightarrow 0$ <br>
funded by the Posts and Telecomm<br>  $0 \rightarrow a \, M \rightarrow M \rightarrow H_0^{I,J}(M) \rightarrow 0$ <br>
funded by the Posts and Telecomm<br>  $\otimes_R a \, M \rightarrow R/\mathfrak{a} \otimes_R M \rightarrow R/\mathfrak{a} \otimes_R H_0^{I,J}(M) \rightarrow 0$ .<br>
REFERENCES<br>
11 plants that  $\alpha N \rightarrow M \rightarrow M + H_0^{I,J}(M) \rightarrow 0$ <br>
induces the following exact sequence<br>  $R/\mathfrak{a} \otimes_R \mathfrak{a} M \rightarrow R/\mathfrak{a} \otimes_R M \rightarrow R/\mathfrak{a} \otimes_R H_0^{I,J}(M) \rightarrow 0$ .<br>
R/ $\mathfrak{a} \otimes_R \mathfrak{a} M \rightarrow R/\mathfrak{a} \otimes_R H_0^{I,J}(M)$  is CFA.<br>
Let  $t > 0$ . Since  $H$  $I \to R/\mathfrak{a} \otimes_R M \to R/\mathfrak{a} \otimes_R H_0^{I,J}(M) \to 0.$ <br>  $\mathfrak{a} \otimes_R H_0^{I,J}(M)$  is CFA.<br>  $\therefore$  Since  $H_t^{I,J}(M) \cong H_t^{I,J}(\cap \mathfrak{a} M) = \text{Comp}$ <br>
we can replace M by  $N = \mathfrak{a} M$ . It im-<br>  $\mathfrak{a} \vee N = N$ . Hence, there is an element<br>
hat  $R/\mathfrak{a}\otimes_R \mathfrak{a} M \to R/\mathfrak{a}\otimes_R M \to R/\mathfrak{a}\otimes_R H_0^{I,J}(M) \to 0.$ <br>
By [11],  $R/\mathfrak{a}\otimes_R H_0^{I,J}(M)$  is CFA.<br>
Let  $t > 0$ . Since  $H_t^{I,J}(M) \cong H_t^{I,J}(\cap \mathfrak{a} M) =$ <br>  $H_t^{I,J}(\mathfrak{a} M)$ , we can replace  $M$  by  $N = \mathfrak{a} M$ . It im-<br>
pl

$$
0 \to 0 :_N x \to N \xrightarrow{x} N \to 0
$$

\n plies that 
$$
\mathfrak{a} N = N
$$
. Hence, there is an element  $x \in \mathfrak{a}$  such that  $xN = N$ . The short exact sequence\n  $0 \rightarrow 0:_{N} x \rightarrow N \xrightarrow{y} N \rightarrow 0$ \n

\n\n induces a long exact sequence  $\cdots \rightarrow H_{t}^{I,J}(N) \xrightarrow{y} H_{t}^{I,J}(N) \xrightarrow{y} H_{t-1}^{I,J}(0:_{N} x) \rightarrow$ \n

\n\n The first term is  $\mathfrak{a} \cup \mathfrak{a} \cup \mathfrak{a}$ .\n

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\n\n The first term is  $\mathfrak{a} \cup \mathfrak{a}$ .\n

\n\n The first term is <

By the assumption,  $H_i^{I,J}(0:_{N} x)$  is CFA for all  $\Rightarrow H_{t-1}^{I,J}(N) \rightarrow \cdots \rightarrow H_0^{I,J}(N) \rightarrow 0.$ <br>By the assumption,  $H_i^{I,J}(0 :_N x)$  is CFA for all<br> $i < t - 1$ . The exact sequences<br> $H_t^{I,J}(N) \xrightarrow{x} H_t^{I,J}(N) \rightarrow \text{Im }\alpha \rightarrow 0$ <br>and<br> $0 \rightarrow \text{Im }\alpha \rightarrow H_{t-1}^{I,J}(0 :_N x) \rightarrow \text{Im }\beta \rightarrow 0$ <br>lead the isomorphism<br>

$$
H_t^{I,J}(N) \xrightarrow{x} H_t^{I,J}(N) \to \text{Im}\,\alpha \to 0
$$

and

$$
0 \to \operatorname{Im} \alpha \to H_{t-1}^{I,J}(0:_N x) \to \operatorname{Im} \beta \to 0
$$

$$
R/\mathfrak{a}\mathop{\otimes} H_t^{I,J}(N)\cong R/\mathfrak{a}\mathop{\otimes} \operatorname{Im}\alpha
$$

$$
H_t^{I,J}(N) \xrightarrow{x} H_t^{I,J}(N) \to \text{Im }\alpha \to 0 \qquad \text{and}
$$
  
\nand  
\n
$$
0 \to \text{Im }\alpha \to H_{t-1}^{I,J}(0:_N x) \to \text{Im }\beta \to 0 \qquad \text{Mac}
$$
  
\nlead the isomorphism  
\n
$$
R/\mathfrak{a} \otimes H_t^{I,J}(N) \cong R/\mathfrak{a} \otimes \text{Im }\alpha \qquad \text{for}
$$
  
\nand the long exact sequence  
\n
$$
\text{Tor}_1^R(R/\mathfrak{a}, \text{Im }\beta) \to R/\mathfrak{a} \otimes \text{Im }\alpha \to \text{for}
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$$
\to R/\mathfrak{a} \otimes H_{t-1}^{I,J}(0:_N x).
$$
 Take

 $\begin{array}{lllllllllllllll} 0\to \mbox{Im}\,\alpha\to H^{I,J}_{t-1}(0:_Nx)\to \mbox{Im}\,\beta\to 0 & & & \mbox{Macdonald I. G. (1962). D} \\ & \mbox{lead the isomorphism} & & \mbox{Macdonald I. G. (1973).} \end{array} \begin{array}{lllllllllllllll} \end{array}$  R/ $\frak{a}\otimes H^{I,J}_t(N)\cong R/\frak{a}\otimes \mbox{Im}\,\alpha & & \mbox{tation of modules over a } \\ & \mbox{Equation of modules over a } Symposia Mathematica 11 \\ & \mbox{and the long exact sequence} & &$ lead the isomorphism<br>  $R/\mathfrak{a} \otimes H_t^{I,J}(N) \cong R/\mathfrak{a} \otimes \text{Im }\alpha$ <br>
and the long exact sequence<br>  $\text{Tor}_1^R(R/\mathfrak{a}, \text{Im }\beta) \to R/\mathfrak{a} \otimes \text{Im }\alpha \to$ <br>  $\to R/\mathfrak{a} \otimes H_{t-1}^{I,J}(0:_N x).$ <br>
It follows from (Tri, 2021) that  $\text{Im }\beta$  is CF fore, so is  $\text{Tor}^R_1(R/\mathfrak{a}, \text{Im }\beta)$ . The inductive hypoth- $\begin{array}{lllllllllllll} \mbox{phism} & \mbox{Macdonald I. G. (1973).} \\ \otimes H_t^{I,J}(N) \cong R/\mathfrak{a} \otimes \operatorname{Im}\alpha & \mbox{fational d. C. (1973).} \\ \mbox{factor sequence} & \mbox{Rotman J. J. (2009).} \hbox{ } An \mbox{d}n & \mbox{mological algebra, Spring} \\ \mbox{R/\mathfrak{a},\operatorname{Im}\beta)} \rightarrow R/\mathfrak{a} \otimes \operatorname{Im}\alpha \rightarrow & \mbox{mological algebra, Spring} \\ \mbox{R/\mathfrak{a} \otimes H_{t-1}^{I,J}($  $R/\mathfrak{a} \otimes H_t^{I,J}(N) \cong R/\mathfrak{a} \otimes \text{Im }\alpha$ <br>
and the long exact sequence<br>  $\text{Tor}_1^R(R/\mathfrak{a}, \text{Im }\beta) \to R/\mathfrak{a} \otimes \text{Im }\alpha \to$ <br>  $\to R/\mathfrak{a} \otimes H_{t-1}^{I,J}(0:_N x).$ <br>
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and the long exact sequence<br>  $\text{Tor}_1^R(R/\mathfrak{a}, \text{Im }\beta) \to R/\mathfrak{a} \otimes \text{Im }\alpha \to \text{modgical algebra, Springer I}$ <br>  $\to R/\mathfrak{a} \otimes H_{t-1}^{t, J}(0:_{N} x).$ <br>
It follows from (Tri, 2021) and the long exact sequence<br>  $\text{Tor}_1^R(R/\mathfrak{a}, \text{Im }\beta) \to R/\mathfrak{a} \otimes \text{Im }\alpha \to$ <br>  $\to R/\mathfrak{a} \otimes H_{t-1}^{I,J}(0:_N x).$ <br>
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It follows from (Tri, 2021) that  $\text{Im }\beta$  is CFA. The<br>
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because the research of a GFA module. Therefore, we have  $R/\mathfrak{a} \otimes \text{Im }\alpha$  is CFA and this Yassemi S. (1995).<br>
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3 CONCLUSION Zöschinger H. (1983). I uln über Noethersche<br>
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of these modules and gave a result on the finitness<br>of these modules and gave a result on the finitness<br>of coassociated primes of these modules.<br>Acknowledgements. This work was supported and

 $0 \to \mathfrak{a} M \to M \to H_0^{1,0}(M) \to 0$  tude of Technology (PT  $Do Ngoc \text{ Yen/Vol } 10. \text{ No } 4\_August 2024$ <br>
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