

**ẢNH HƯỞNG CỦA LASER LIÊN KẾT BĂNG RỘNG
ĐỐI VỚI TRONG SUỐT CẢM ỨNG ĐIỆN TỪ CỦA HỆ KIỂU Λ VỚI CẤU TRÚC FANO**

**Influence of broadband coupling laser
on electromagnetically induced transparency of Λ -like system with the Fano structure**

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TÓM TẮT

Trong suốt cảm ứng điện từ cho hệ kiểu Λ bao gồm hai trạng thái giới hạn dưới và một liên tục phẳng liên kết với hai trạng thái tự ion hóa được gọi là liên tục Fano đôi gắn vào trong nó, trong đó laser liên kết được mô hình hóa bởi nhiễu trắng đã được nghiên cứu. Đối với hệ chứa các mức tự ion hóa rời rạc như thế chúng tôi tìm được hệ các phương trình vi tích phân ngẫu nhiên liên kết có thể được lấy trung bình chính xác. Từ đó tìm được biểu thức chính xác xác định nghiệm dừng đối với độ cảm điện. Phổ thành phần tán sắc và hấp thụ của trong suốt cảm ứng điện từ đã tìm được và so sánh chúng với những kết quả thu được trước đó bởi chúng tôi và các tác giả khác.

Từ khóa: *Trong suốt cảm ứng điện từ; liên tục Fano đôi; cấu hình Λ ; nhiễu trắng.*

ABSTRACT

Electromagnetically induced transparency for Λ -like system consisting of two lower bound states and a flat continuum coupled to two autoionization states, it is so-called the double Fano continuum, embedded in it is studied in which the coupling laser is modeled by white noise. For such a system containing discrete autoionization levels we obtain a set of coupled stochastic integro-differential equations which can be averaged exactly. This leads to the exact expression determining the stationary solution for the electric susceptibility. The spectra of dispersion and absorption components for electromagnetically induced transparency are found and compared with those derived previously by us and other authors.

Keywords: *Electromagnetically induced transparency; double Fano continuum; Λ configuration; white noise.*

1. Introduction

Laserlight is never perfectly monochromatic, it is generally fluctuating in amplitude and phase. The microscopic natural world is extremely complex, so we cannot research it directly but must model it by the classical stochastic processes, which are time

dependent. All current stochastic models of the laser have a common character: the laser is a stationary Gaussian stochastic process with the finite correlation time. Exact analytical averaging of stochastic equations with Gaussian noise with finite correlation time is a difficult task. Practically only the extreme case of white

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noise has been well researched. The model of the white noise for the field is interesting by itself because it describes the electric field amplitude of the multimode laser, operating without any correlation between the modes.

Electromagnetically induced transparency (EIT) phenomenon relies on the destructive quantum interference of the transition amplitudes involved in the system that leads to the suppressing of the absorption or even the complete transmission of the resonant weak probe beam in the presence of the second strong coupling laser beam [1-4]. This phenomenon has been observed in various experiments for three basic configurations of a three-level system: Λ -, V-, and ladder [5,6] (for the ladder configuration [7], Λ configuration [8] with extension to number of lower levels more than two, referred to as tripod ones [9]). This effect has potential applications in the protocols that create quantum memory for quantum computers [10]. EIT in a model Λ -like system consisting of two lower bound states and a continuum coupled to an autoionization (AI) state embedded in it has been considered in [11]. The latter state might also be due to an interaction with an additional laser. The authors obtained analytic expressions for the susceptibility in the case of the bound-continuum dipole matrix elements being modelled according to Fano autoionization theory [12] and examined the shape of the transparency window depending on the amplitude of the control field.

Recently the model studied in [11] has been extended to the case where the continuum involved in the problem is replaced by one with so-called the double- Λ system [13] or double Fano structure [14], where instead of one AI state we have two AI states with the same energy [13] or two discrete AI states [14] embedded in the continuum. It has

been shown that the presence of the second AI level leads to the additional EIT window appearance. In this paper we use the same method applied in [15,16] to consider the model that studied in [14] by modeling of fluctuating control field as a white noise. Then the set of coupled stochastic integro-differential equations involved in the problem can be also solved exactly. The spectra of real and imaginary parts of the medium susceptibility are calculated and compared with the results obtained before by us and other authors. It follows that the structure of the EIT windows changes dramatically when the control field fluctuates.

2. The model

As shown in figure 1, we consider the Λ system that includes two lower states $|b\rangle$ and $|c\rangle$, two autoionizing states $|a_1\rangle$ and $|a_2\rangle$, and the bare continuum $|E\rangle$. This continuum is coupled with the states $|a_1\rangle$ and $|a_2\rangle$ by two additional couplings U_1 and U_2 , respectively. By using the diagonalization method of the Fano [12] we can replace all excited states with a dressed continuum $|E\rangle$ that is so-called the double Fano continuum [17-19]. The state $|b\rangle$ is coupled to the $|E\rangle$ by a weak probe laser with the frequency ω_p and amplitude ε_p , whereas the state $|c\rangle$ is coupled to the $|E\rangle$ by a strong control laser with the frequency ω_d and amplitude ε_d . As usual in the works concerning autoionization phenomena, for convenience we assume that the frequency ω_d is not large enough to allow for the transition from the state $|b\rangle$ to the continuum and omit level shifts due to nonresonant couplings, which can be taken into account by redefining involved detuning.

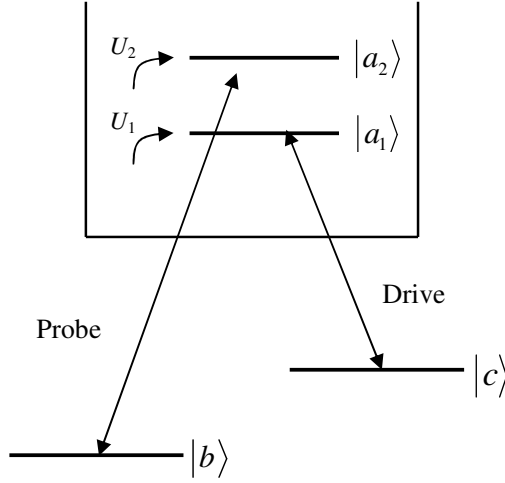


Figure 1: The level and coupling scheme.

Applying the formalism of Fano diagonalization the differential equations for the matrix elements of statistical operator and averaging of these equations, we obtain the system of equations for stochastic averages of the variables in the form:

$$\begin{aligned} i\hbar\dot{\rho}_{ab} &= \left[(E - E_b - \hbar\omega_p) + \frac{a_0}{8\Gamma} \langle E|d|c\rangle\langle c|d|E\rangle \right] \rho_{ab} - \frac{1}{2} \langle E|d|b\rangle \varepsilon_p - \frac{1}{2} \langle E|d|c\rangle b_0 \rho_{cb}, \\ i\hbar\dot{\rho}_{cb} &= \left[(E_c + \hbar\omega_d - E_b - \hbar\omega_p - i\hbar\gamma_{cb}) + \frac{a_0}{8\Gamma} \langle c|d|E\rangle\langle E|d|c\rangle \right] \rho_{cb} - \frac{1}{2} b_0^* \langle c|d|E\rangle \rho_{ab} dE, \end{aligned} \quad (1)$$

where b_0 is a deterministic coherent part and a_0 is fluctuation part of the driving field. By using the solution of these equations we can get the density matrix elements necessary for determination of the medium susceptibility and spectra of them.

3. The susceptibility spectrum

We can use the following relation

$$P^+(\omega_p) = N \int d_{bE} \rho_{Eb} dE = \varepsilon_0 \varepsilon_1 \chi(\omega_p), \quad (2)$$

to obtain the spectrum of the susceptibility $\chi(\omega_p)$ from the density matrix elements, in which ε_0 and N are the vacuum electric permittivity and the atom density, respectively. Thus, the medium susceptibility χ has form:

$$\chi(\omega_p) = -\frac{N}{\varepsilon_0} \left(R_{ab} + \frac{\frac{1}{4} b_0^2 R_{bc}^* R_{cb}}{E_b + \hbar\omega_p - E_c - \hbar\omega_d + i\hbar\gamma_{cb} - \frac{1}{4} b_0^2 R_{cc}} \right). \quad (3)$$

The functions $R_{jk}(\omega_p)$ and $R'_{jk}(\omega_p)$, $j, k = b, c$ appearing in (3) have the form:

$$R_{jk}(\omega_p) = \lim_{\eta \rightarrow 0^+} B_j B_k \int \frac{F_j(E) F_k(E)}{E_b + \hbar\omega_p - E - \frac{a_0}{8\Gamma} B_c^2 |F_c(E)|^2 + i\eta} dE, \quad (4)$$

$$R'_{jk}(\omega_p) = \lim_{\eta \rightarrow 0^+} B_j B_k \int \frac{F_j(E) F_k(E)}{\left(E_b + \hbar\omega_p - E - \frac{a_0}{8\Gamma} B_c^2 |F_c(E)|^2 + i\eta \right) \left(1 + \frac{\frac{a_0}{8\Gamma} B_c^2 |F_c(E)|^2}{E_c + \hbar\omega_d - E_b - \hbar\omega_p - i\hbar\gamma_{cb} + (1/4) b_0^2 R_{cc}} \right)} dE. \quad (5)$$

in which the limit $\eta \rightarrow 0^+$ assures that $\text{Im } \chi > 0$, and

$$\begin{aligned} F_j(E) &= (Q_j + i) \left(\frac{1}{Q_j + i} + \frac{A_j^+}{E - E_+} + \frac{A_j^-}{E - E_-} \right), \\ F_k(E) &= (Q_k - i) \left(\frac{1}{Q_k - i} + \frac{(A_j^+)^*}{E - (E_+)^*} + \frac{(A_j^-)^*}{E - (E_-)^*} \right). \end{aligned} \quad (6)$$

In formula (6) E_{\pm} given by formula

$$E_{\pm} = \frac{E_1 + E_2 \pm \eta}{2} + i \frac{\gamma \pm \phi}{2}, \quad (7)$$

in which

$$\begin{aligned} \phi &= \frac{1}{\sqrt{2}} \left\{ \left[(E_{21}^2 - \gamma^2)^2 + 4E_{21}^2 (\gamma_2 - \gamma_1)^2 \right]^{1/2} - E_{21}^2 + \gamma^2 \right\}^{1/2}, \\ \eta &= \frac{1}{\sqrt{2}} \left\{ \left[(E_{21}^2 - \gamma^2)^2 + 4E_{21}^2 (\gamma_2 - \gamma_1)^2 \right]^{1/2} + E_{21}^2 - \gamma^2 \right\}^{1/2}, \end{aligned} \quad (8)$$

The complex amplitudes A_j^{\pm} are given by the following formula:

$$A_j^{\pm} = \frac{\gamma}{2} \left(1 \pm \frac{E_{21} K_j + i\gamma}{\eta + i\phi} \right), \quad (9)$$

in which

$$K_j = \frac{Q_{j21} + i\gamma_{21}}{Q_j + i}, \quad (10)$$

where $E_{21} = E_2 - E_1$ is the separation between two autoionizing levels, and effective asymmetry parameters Q_j , Q_{j21} are given by

$$Q_j = \frac{q_{1j}\gamma_1 + q_{2j}\gamma_2}{\gamma}, \quad Q_{j21} = \frac{q_{2j}\gamma_2 - q_{1j}\gamma_1}{\gamma}, \quad (11)$$

where $\gamma_1 = \pi \langle a_1 | U_1 | E \rangle^2$ and $\gamma_2 = \pi \langle a_2 | U_2 | E \rangle^2$ are autoionizing widths involved in the system. Moreover, similarly as in [17], we have used Fano asymmetry parameters q_{1j} and q_{2j} which are given by:

$$q_{1j} = \frac{\langle j | d | a_1 \rangle}{\pi \langle j | d | E \rangle \langle E | U | a_1 \rangle}, \quad q_{2j} = \frac{\langle j | d | a_2 \rangle}{\pi \langle j | d | E \rangle \langle E | U | a_2 \rangle}, \quad (12)$$

and γ_{21} has the form

$$\gamma_{21} = \frac{\gamma_2 - \gamma_1}{\gamma}. \quad (13)$$

Moreover, The matrix elements of the dipole moment transition $\langle j | d | E \rangle$ and $\langle E | d | k \rangle$ are denoted by B_j and B_k , respectively.

As it was mentioned in previous section, we can extend the lower limit of the integral for $R_{jk}(\omega_p)$ and $R'_{jk}(\omega_p)$ to minus infinity. Thus the formulae (4) and (5) can be computed completely numerically. After substituting them into formula (3), we have found the susceptibility $\chi(\omega_p)$ in the stationary regime.

By assuming the same values for the parameters describing the atomic system and its interaction with external fields, we easily compare our results with those in [11, 13-16]. Thus, we have assumed that the total autoionization rate $\gamma = 10^{-9} a.u.$, the coupling constants are $B_b = 2 a.u.$, and $B_c = 3 a.u.$ The value of coherent component b_0 is in range from 10^{-9} to $10^{-6} a.u.$ (all parameters that used here are in atomic units). Moreover, the asymmetry parameters are of the order of 10-100, γ_{cb} is neglected, and the atomic density is $N = 0.33 \times 10^{12} \text{ cm}^{-3}$. We assume that the parameters that describe autoionizing levels are identical ($\gamma_{21} = 0$ and $Q_{j21} = 0$). The detuning ω occurring in the figures is given by $\omega = \omega_p + (E_b - E_1) / \hbar$.

The spectra of real and imaginary parts of the medium susceptibility for various values of the parameters involved in the problem are shown in figures.

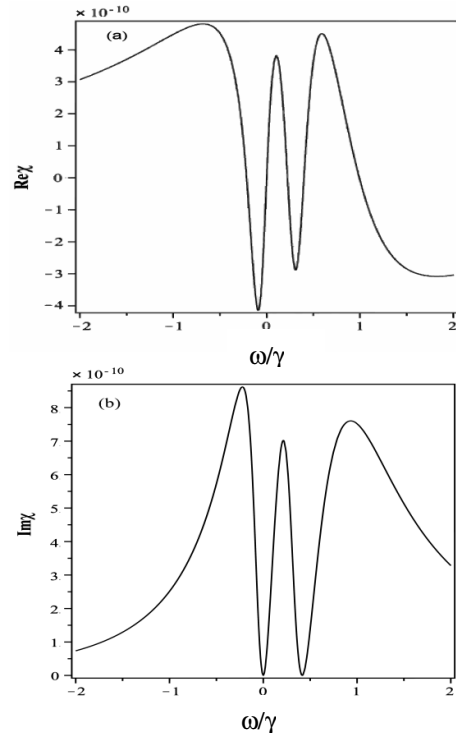


Figure 2: The real (a) and imaginary (b) parts of the susceptibility as a function of the ω/γ for the value of $b_0 = 4 \times 10^{-7} a.u.$, $\gamma = 10^{-9} a.u.$, and $Q_b = Q_c = 10$, $E_{21} = 0.8\gamma$, and the fluctuation part $a_0 = 0$.

When coherent component of the laser light dominates over the fluctuations, we can assume that the fluctuation component of the field amplitude vanishes ($a_0 = 0$) and then, our result becomes exactly the same as that obtained in [13] and the spectra of real and imaginary parts of the medium susceptibility for these cases are shown in the figure 2. Actually, for the case ($E_1 = E_2$) we get the same result as for the model with a single AI level discussed in [11]. Moreover for the case $E_{21} \rightarrow 0$, our result becomes exactly the same as that obtained by Thuan Bui Dinh *et al.* [13] (Fig. 3).

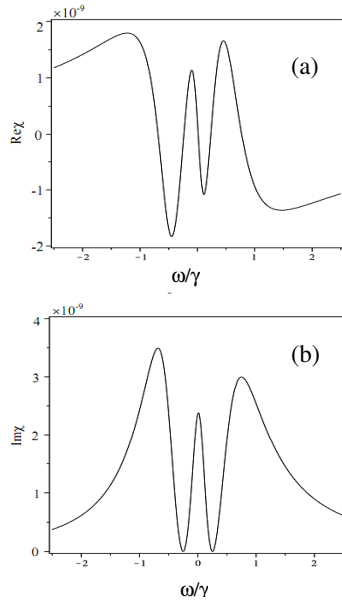


Figure 3: The real (a) and imaginary (b) parts of the susceptibility as a function of the ω/γ for the value of $b_0 = 4 \times 10^{-7} a.u.$, $\gamma_{21} = 0$, $Q_b = Q_c = 20$, $Q_{b21} = 1$, $Q_{c21} = 8$ and $a_0 = 0$.

However, for the general case, not only the fluctuation component of the control field amplitude is present but also coherence component plays significant role. If AI levels are of equal energy ($E_{21} = 0$) then our result becomes the same as that obtained in [15]. In addition, when $E_{21} \rightarrow 0$, we get the same result as for the model with two AI levels of the same energy derived by Doan Quoc Khoa *et al.* [16]. These have been already studied in [15,16]. The real and imaginary components of the electric susceptibility for these cases are shown in the figures 4 and 5. For the general case ($E_1 \neq E_2$), the real and imaginary parts of the electric susceptibility are showed in the figures 6 and 7. When the fluctuation part is absent, these become exactly with that discussed in [14] (Fig.6). If fluctuation component is present then the spectrum of real and imaginary parts of the medium susceptibility (Fig.7) also contains two transparency windows but the slope of the dispersion curve and absorption profiles decrease and the zero point shifts to the right

in comparison with the case when the chaotic component is absent.

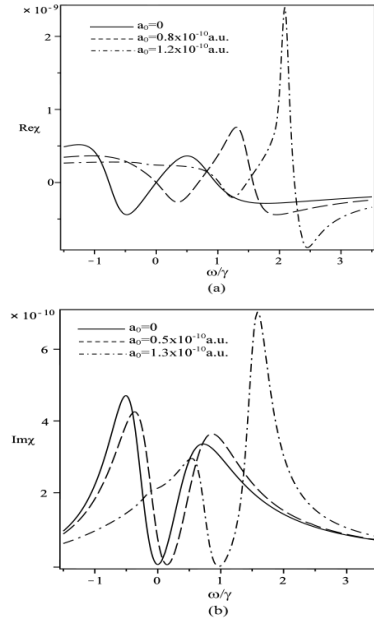


Figure 4: The real (a) and imaginary (b) parts of the susceptibility as a function of the ω/γ for various values of a_0 , the coherent part $b_0 = 10^{-6} a.u.$, the Fano parameters are $q_b = q_c = 10$ (a) and $q_b = q_c = 7$ (b).

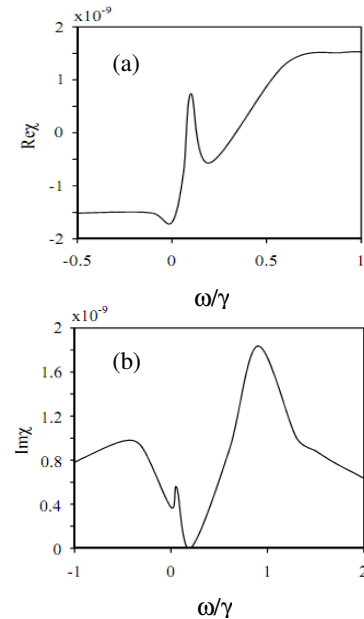


Figure 5: The real (a) and imaginary (b) parts of the susceptibility as a function of the ω/γ for the value of $b_0 = 4 \times 10^{-7}$, $\gamma_{21} = 0$, $Q_b = Q_c = 20$, $Q_{b21} = 1$, $Q_{c21} = 8$ and $a_0 = 0.02\gamma$.

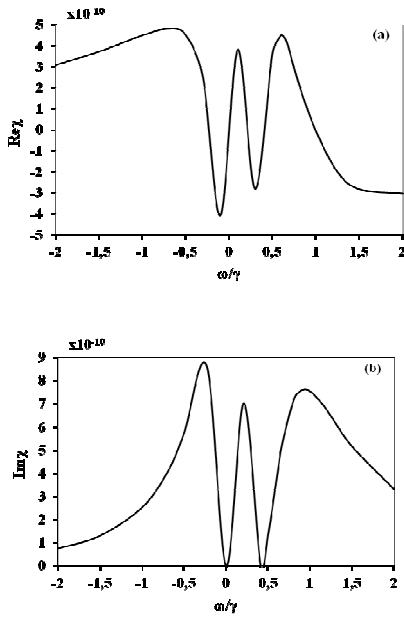


Figure 6: The real (a) and imaginary (b) parts of the susceptibility as a function of ω/γ and the coherent part $b_0 = 4 \times 10^{-7}$ a.u., $\gamma = 10^{-9}$ a.u., $Q_b = Q_c = 10$, $E_{21} = 0.8\gamma$ and $a_0 = 0$.

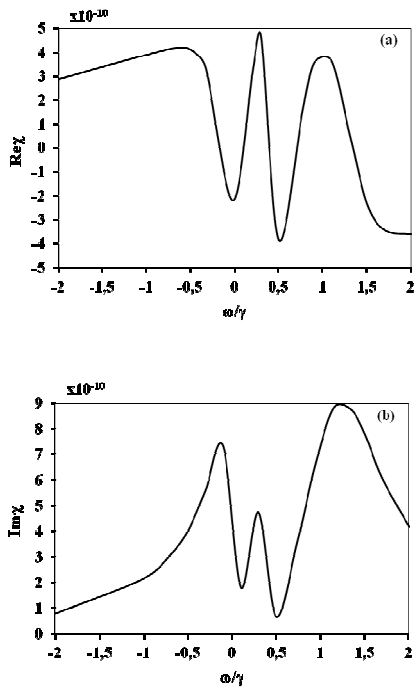


Figure 7: The real (a) and imaginary (b) parts of the susceptibility as a function of ω/γ and the coherent part $b_0 = 4 \times 10^{-7}$ a.u., $\gamma = 10^{-9}$ a.u., $Q_b = Q_c = 10$, $E_{21} = 0.8\gamma$ and $a_0 = 0.0025\gamma$.

4. Conclusions

In this paper we discussed the atomic model of double Fano continuum that considered in [14]. We assumed that, the coupling laser light applied in the system is modeled by white noise. For such a system the stationary solution for the electric susceptibility was obtained by solving a set of coupled stochastic integro-differential equations involved in the problem. Next, we derived the exact formulas determining the spectra of real and imaginary parts of the medium susceptibility and compare them with that derived in [14]. We have shown that, similarly as in [14], the EIT effect appears for the system discussed here. Moreover, both the position and the width of the transparency window of this effect change dramatically as we compare them with those in the case of the noise of the control laser field is absent. Similarly to the case considered in [15,17], we believe that our model is more realistic than that studied in [13, 14], because the amplitudes of the real laser used in experimental setups always contain some fluctuating component.

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