Calculation of the Ettingshausen coefficient in quantum wells with parabolic potential in the presence of electromagnetic wave (for electron-confined acoustic phonons scattering)

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Abstract

By using the quantum kinetic equation for the distribution function of electrons, the expression for Ettingshausen coefficient (EC) in quantum wells with parabolic potential (QWPP) in the presence of electromagnetic wave (EMW) is obtained for electrons - confined acoustic phonons scattering. The analytic results have shown that EC depends on temperature, magnetic field, characteristic quantities of EMW and $m$ - quantum number which is specific the confined phonons in a complicated way. The numerical results for GaAs/GaAsAl quantum wells (QW) have displayed these dependence explicitly. In particular, when $m$ is set to zero, we achieve results for magneto–thermoelectric effect in the same QW without the confinement of acoustic phonons.

1. Introduction

Both wave function and energy spectrum of the electrons are quantized under the influence of confinement effect. So, the low-dimensional semiconductor systems (LDSS) have not only changed physical properties but also being appeared new effects [1-5]. Among them, we have to mention Ettingshausen effect. That is a thermoelectric phenomenal that effects the current in conductor in the presence of magnetic field. The creation of electron-hole pairs at one side and their recombination at the other side of the sample are the main cause of Ettingshausen effect in semiconductors [6]. This effect was also studied some twodimensional semiconductor systems [3,4]. However, those studies have not interested in the confinement of phonons. In other hand, several examinations have shown that the confined phonons significant influence on quantum effects in LDSS: confined LO-phonons create new properties of the Hall effect in doped semiconductor superlattices [1]; confined optical phonons makes a remarkable impact on the Hall effect [2] and increase the number of resonance peaks of the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons [5] in a compositional superlattices. So far, how the CAP influence on the Ettingshausen effect in QWPP is still an unanswered question.

In this work, a QWPP in the presence of constant electric field, magnetic field and EWM have been considered for Ettingshausen effect [3]. We have taken electron-CAP scattering into account and obtained analytic expression for the EC. In the process of transformation, we always count on the temperature gradient.
Components of the article are as follows: In section 2, we get the analytic equation of the EC based on computation related to the Hamiltonian of electron. We give the result of numerical calculation and discussion in section 3. Final section contains conclusions.

2. The Ettingshausen coefficient in the QWPP under the influence of confined acoustic phonons

We have considered a QW with parabolic potential: 

\[ V(z) = m \omega^2 z^2 / 2 \]  

(with \( m \) is detention frequency characteristic QWPP). There exists a magnetic field with \( \mathbf{B} = (0, 0, B) \) and constant electric field with \( \mathbf{E} = (E_x, 0, 0) \). In this case, the movement of electrons is limited to \( Oz \); so, they can only move freely in the \( x-y \) plane with cyclotron frequency \( \omega_c = eB / m_e \) and imply velocity \( \mathbf{v} = E_x / B \).

That means QWPP have been considered in the condition: the magnetic field is perpendicular to the free-moving plane of electrons. Energy of an electron is and being received intermittent values:

\[ eV_n(a_p) = \left[ N + \frac{1}{2} \right] \hbar \omega_c + \left( n + \frac{1}{2} \right) \hbar \omega + \frac{m \nu^2}{2} - \hbar v_c p_y \]  

(2.1)

Here \( p_y \) is the wave vector of electrons in the \( y \)-direction.

When QWPP is subjected to a laser radiation \( E_0(t) = E_0 \sin(\Omega t) \). Hamiltonian of the electron CAP system can be expressed as:

\[ H = \sum_{n,s} \varepsilon_{n,s} \left( p_y - \frac{e}{\hbar c} A(t) \right) a_{n,s}^+ a_{n,s} \]

\[ + \sum_{n,s} \left( \sum_{n',s'} D_{n,n',s,s'}(q) a_{n',s'}^+ a_{n,s} - \right) \]

\[ + \sum_{q} \varphi(q) a_{n,q}^+ a_{n,q} \]

(2.2)

In which: \( a_{n,s}^+ \), \( a_{n,s} \) are the creation and annihilation operators of electrons (phonons) respectively; \( A(t) = \frac{e}{\Omega} E_0 \cos(\Omega t) \) is the vector potential of laser field; \( \varphi(q) = \frac{1}{(2\pi)^3} \left\{ e^{i \mathbf{E}_0 + \pi \hbar \mathbf{v} \cdot \mathbf{q} / \hbar} \right\} \) is energy of a CAP with the wave vector \( q = (q_x, q_y) \) and \( q = q_x + q_y \); \( m \) is the detention index of phonons.

\[ D_{n,N,n',N',s,s'}(q) = \left| C_n(q) \right|^2 \left| J_{n,n'}(u) \right|^2 \]

\[ L = \frac{1}{L^2} \int e^{i \mathbf{p} \cdot \mathbf{u}} e^{i \mathbf{p} \cdot \mathbf{q}} H_n \left( \frac{\mathbf{z}}{L} \right) \overline{H_n} \left( \frac{\mathbf{z}}{L} \right) d\mathbf{z} \]

is the electron form factor.

\[ J_{n,N}(u) = \frac{N_{\text{min}}}{N_{\text{max}}} \cdot e^{-u} u^{N_{\text{max}} - N_{\text{min}}} \left[ L_{N_{\text{min}}}^{N_{\text{max}} - N_{\text{min}}}(u) \right]^2 \]

with \( L_N^{N_{\text{min}}}(u) \) is the associated Laguerre polynomial.

The quantum kinetic equation of average number of electron is:

\[ i\hbar \frac{\partial f_{n,s}(t)}{\partial t} = \left\{ a_{n,s}^+ a_{n,s}, H \right\} \]

(2.3)

in which
Using (2.2) for (2.3) then we performed transformations of operator algebra and obtained:

\[
\begin{align*}
\frac{\partial^2 f_{x,y}(t)}{\partial p_y} &+ \left[E_y + \mu_{x,y}(p_y)\right] \frac{\partial f_{x,y}}{\partial p_y} \\
&= \sum_{k \neq \emptyset} D_{s \times s',\emptyset}(\Omega) \sum_{s \neq \emptyset} \frac{d}{dt} \left[ \frac{1}{2} \right] \\
&* \left[ \begin{array}{c}
\left[ f_{s \times s',\emptyset}((N_{x,y} - 1)^{-1} - f_{s \times s',\emptyset}((N_{x,y} - 1)^{-1} + \text{h}o_{x,y} + h\Omega) \\
\left[ f_{s \times s',\emptyset}((N_{x,y}^{-1}) - f_{s \times s',\emptyset}((N_{x,y}^{-1} + \text{h}o_{x,y} + h\Omega) \\
\end{array} \right]
\end{align*}
\]

where:

\[
\lambda = \frac{eE_0 q_y}{m \epsilon^2}; N_{m,q_{\perp}} = b^+ b \rightarrow m_{q_{\perp}} m_{q_{\perp}}
\]

is the equilibrium distribution function of the phonons.

For simplicity, we limit to the case of \( l = 0, \pm 1 \), get to close

\[
J_0^2 \left( \frac{\lambda}{\Omega} \right) = 1 - \frac{1}{2} \left( \frac{\lambda}{\Omega} \right)^2; J_1^2 = \frac{1}{4} \left( \frac{\lambda}{\Omega} \right)^2
\]

. We multiply both sides by (2.4) with \( \frac{e}{m} p_y \delta \left( e - \epsilon_{N,q} \left( p_y \right) \right) \) then taking sum of \( N, n, \) and \( p_y \). We get following expression:

\[
\tilde{G}(\epsilon) + w_{\epsilon} \left[ \tilde{h}, \tilde{G}(\epsilon) \right] = \tilde{P}(\epsilon) + \tilde{Z}(\epsilon)
\]

(2.5)

In the above expression, we use symbols to replace complex equations. \( \tilde{h}, \tilde{G}(\epsilon) \) directional multiplication of \( \tilde{h} \) and \( \tilde{G}(\epsilon) \).

\[
\tilde{P}(\epsilon) = -\frac{e}{m} \sum_{N,q_{\perp}} p_y \tilde{F} \frac{\partial^2 f_{N,q_{\perp}}}{\partial p_y} \delta \left( e - \epsilon_{N,q} \left( p_y \right) \right)
\]

(2.6)

And

\[
\tilde{Z}(\epsilon) = \frac{e}{m} \sum_{N,q_{\perp}} p_y \tilde{F} \frac{\partial^2 f_{N,q_{\perp}}}{\partial p_y} \delta \left( e - \epsilon_{N,q} \left( p_y \right) \right)
\]

(2.7)

with \( \tau \) is the momentum relaxation time and \( \tilde{F} = eE_1 - \frac{e - \epsilon_F}{T} \nabla T \) (\( \epsilon_F \) is the Fermi energy of electron).

By solving the equation (2.5), we find out expression of individual current density:

\[
\tilde{G}(\epsilon) = \tau(\epsilon) \left[ 1 + \frac{\epsilon^2}{\epsilon^2} (\epsilon) \right] \tilde{P}(\epsilon) + \tilde{Z}(\epsilon)
\]

\[
\tilde{Z}(\epsilon) = \tau(\epsilon) \\
- w_{\epsilon} (\epsilon) \left[ \tilde{h}, \tilde{P}(\epsilon) \right] + \tilde{h}, \tilde{Z}(\epsilon)
\]

(2.8)

The total current density \( \tilde{J} \) and the thermal flux density \( \tilde{Q} \) are given by:

\[
\tilde{J} = \int_0^\infty \tilde{G}(\epsilon) \, d\epsilon
\]

(2.9)

And

\[
\tilde{Q} = \int_0^\infty \left( \epsilon - \epsilon_F \right) \tilde{G}(\epsilon) \, d\epsilon
\]

(2.10)

In low temperature conditions, the electron gas in QW is completely degenerated. The equilibrium distribution function of electron is of the form:

\[
f^0_{N,q_{\perp}} = n_{\theta} \left( \epsilon_F - \epsilon_{N,q_{\perp}} \right)
\]

The distribution function of electron is found in linear approximation by:

\[
f_{N,q_{\perp}}(\epsilon) = f^0_{N,q_{\perp}} - \frac{\epsilon}{m} \tilde{p}_{q_{\perp}} \tilde{p}_{q_{\perp}} \left( \epsilon, \tilde{p}_{q_{\perp}} \right) \frac{\partial f^0_{N,q_{\perp}}}{\partial q_{\perp}}
\]

(2.11)

here:

\[
\tilde{Z}(\epsilon) = \frac{\tau(\epsilon)}{m} \left[ \tilde{F}(\epsilon) - w_{\epsilon}(\epsilon) \left[ \tilde{h}, \tilde{F}(\epsilon) \right] \right]
\]

(2.12)

From expressions of the total current density and the thermal flux density achieved, comparing it to the writing:

\[
J_\rho = \sigma_{\rho} E_\rho + \mu_{\rho} \nabla T \quad \text{and} \quad Q_\rho = \epsilon_{\rho} E_\rho + \phi_{\rho} \nabla T
\]

we obtain analytic expression of tensors:
Here:
\[
\phi_m (m) = \sum_{n' \neq n} \frac{(n-n') \hbar \omega_c + eE \vec{F} \pm \hbar \omega_m \pm \hbar \Omega}{\hbar \omega_c}
\]
\[
\phi_m (m) = \sum_{n' \neq n} \frac{1}{2} \left[ \left( \alpha_{n,n'} \mp \beta_{n,n'} \right) + \sqrt{\alpha_{n,n'}^2 + \beta_{n,n'}^2} \right]
\]

with \( \omega_c = \frac{1}{\hbar} \sum_{n' \neq n} \left( \phi_{m,n'} \mp \phi_{m,n} \right) \), and \( \phi_{m,n} \) are the elementary energy levels. The energy of the CAP is given by:
\[
E_{m,n} = \left\{ (N-n') \hbar \omega_c + eE \vec{F} \pm \hbar \omega_m \pm \hbar \Omega \right\}
\]
In Eq. (2.17),
\[
\sigma_{\alpha}(m) \sigma_{\beta}(m) - \sigma_{\alpha}(m) \sigma_{\beta}(m)
\]
are components of tensors in Eq. (2.13), Eq. (2.14), Eq. (2.15) and Eq. (2.16), respectively; \( K_L \) is the thermal conductivity of phonons. From analytic expressions, we can see that the EC depends in a complicated way on characteristic quantities of EMW (the amplitude \( E_0 \) and the frequency \( \Omega \)), the temperature, the magnetic field, and especially the m-quantum number being specific to the confined phonons. Interesting the energy of CAP
\[
\left( \hbar \omega_m = \hbar \nu, \frac{m \pi}{L} \right)
\]
leads to abundant analytic results and being added to resonance condition in QW. In particular, we get the results in the case of unconfined acoustic phonons when m is set to zero [3]. These dependencies will be clarified in section 3 when we study QWPP of GaAs/GaAsAl.

3. Numerical results and discussions

To get influence of the CAP on the EC in QWPP in the presence of EMW in detail, we consider the QWPP of GaAs/GaAsAl with the parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_e )</td>
<td>0.067 (e is the mass of a free electron)</td>
</tr>
<tr>
<td>( \nu )</td>
<td>( \frac{5378}{m} )</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>( \rho = 5.32 \text{g/cm}^3 )</td>
</tr>
</tbody>
</table>

With
\[
\phi_{\alpha}(m) = \sum_{n' \neq n} \frac{1}{2} \left[ \left( \alpha_{n,n'} \mp \beta_{n,n'} \right) + \sqrt{\alpha_{n,n'}^2 + \beta_{n,n'}^2} \right]
\]
and
\[
\phi_{\beta}(m) = \sum_{n' \neq n} \frac{1}{2} \left[ \left( \alpha_{n,n'} \mp \beta_{n,n'} \right) + \sqrt{\alpha_{n,n'}^2 + \beta_{n,n'}^2} \right]
\]

\( \lambda_{ij} \) is the anti-symmetrical Levi tensor; \( \delta_{ij} \) is the Kronecker delta and i, j, k, l, p correspond the components x, y, z of the Cartesian coordinates.

The expression of the EC is given by:
\[
\mathcal{E} = \frac{1}{H \sigma_{\alpha}(m)} \mathcal{E}_{\alpha}(m) - \sigma_{\alpha}(m) \mathcal{E}_{\beta}(m)
\]
Figure 1. The dependence of the EC on EMW amplitude

Fig.1 describes the dependence of EC on EMW amplitude in two cases: with and without confinement of acoustic phonons at T=5K. The graph indicates that the EC depends clearly on the EMW in low amplitude domain. The EC rises fast and linearly to reach the horizontal line in both cases to be considered in higher amplitude region. We realize that in the high EMW amplitude condition, the EC is almost unchanged when the EMW amplitude increases. Besides, the EC has negative values with unconfined phonons [3] and even confined.

As can be seen from Fig.2, the EC oscillates strongly when the EMW frequency is less than $10^{12}$ Hz. When the EMW frequency increases from $10^{12}$ Hz to $2.0 \times 10^{12}$ Hz the EC has the same value and almost be unchanged in both cases. In this frequency range, both EC peaks and EC peak positions tend upward. The graph also shows that: peaks of the blue line are sideways to the right and be higher than peaks of red line. We can explain those results as follows: the resonance peaks correspond to the condition:

$$h\Omega = (N' - N)\hbar\omega_c + (n' - n)\hbar\omega + eE_0 + \pm \hbar\omega_0$$

or

$$h\Omega = (N' - N)\hbar\omega_c + (n' - n)\hbar\omega + eE_0 + \pm \hbar\omega_0$$

so, when m increases, the resonance peaks tend to shift to higher frequency regions and corresponding to each resonant frequency, the EC has greater value. Meanwhile, the EC always increases when the EMW frequency increases in the same frequency domain as in electron optical phonons scattering [4]. Moreover, in the case of electron acoustic phonons scattering, the EC has negative values. This result is completely opposite to case of electron optical phonons scattering the EC has positive values [4]. Thus, the scattering mechanism not only affects the values but also the variation of the EC under influence of EMW frequency change.

Figure 2. The dependence of the EC on EMW frequency

Figure 3. The dependence of the EC on temperature

Fig.3a indicates that in both cases - with and without the confinement of acoustic phonons - the EC has negative values and be nearly linear when the temperature increases. In particular, when m goes to zero we obtain the results in the same QWPP in the case of unconfined acoustic phonons [3].

The influence of EMW on the EC is displayed clearly in the Fig.3b. In the temperature domain investigated, the EC has greater values within the presence of the EMW and the confinement of acoustic phonons. However, this influence is weak and almost
only causes change in the magnitude of the EC while temperature increases.

In the Fig.4a, we can see oscillations of the EC when magnetic field changes. The graph shows that both lines oscillate and reach resonant point. The blue line (with CAP) not only has more resonance peaks than the red line (without the confinement of acoustic phonons) but peaks of the blue line are also taller than the red line’s. We can easily explain as follows: when acoustic phonons are confined, their wave vector is quantized; both energy and interaction constant depend on quantum number m; so, the resonance condition is affected by m: the larger the value of m received, the more the resonance peaks of EC. That means the confinement of acoustic phonons affect the EC’s changing law under increasing of magnetic field.

Figure 4. The dependence of the EC on magnetic field

The existence of EMW also governs the EC’s law of change. It is displayed in Fig.4b. $E_0$ is appeared in the argument of the Bessel function and not related to the resonance condition. When $E_0 = 0$ resonance peaks are sideways to the left and have greater values in comparison to the case of $E_0 = 4.10^5 (V/m)$. These are different from the case of unconfined acoustic phonons [3].

4. Conclusions

By using the quantum kinetic equation for electron with the presence of invariable electric field, magnetic field and EMW, in this paper, we have calculated the analytic expression of the EC, graphed the theoretical results for GaAs/GaAsAl QWPP. The achievements get show that the formula of EC depends on many quantities, especially the quantum index m specific the confinement of phonons. All of numerical results indicate that the quantum number m have impacted to the EC. The EC values are greater when we carry out the survey within confinement of acoustic phonons. When acoustic phonons are confined, the EC values or absolute values of the EC are 6 to 10 times as much as the EC without confinement of phonons. In addition, the m also affects the resonance condition and makes the appearance of auxiliary resonance. If m goes to zero, the results obtained come back to the case of unconfined phonons and ignored the energy of acoustic phonons [3]. In the comparison with the case of electron–optical phonons scattering [4], a few results we achieved which are completely opposite. That means the scattering mechanism not only affects the values but also the variation of the EC. Finally, we can assert that the confinement of acoustic phonons creates surprising changes of the EC in the QWPP.

Acknowledgments

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REFERENCES


5. Le Thai Hung, Nguyen Vu Nhan, Nguyen Quang Bau (2012), The impact of confined phonons on the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in compositional superlattices, VNU Journal of Science, Mathematics - Physics Vol.28 68-76;


Tính toán hệ số Ettingshausen trong hố lường tử thể parabol khi có mặt sóng điện tử (trường hợp tán xạ điện tử-phonon âm gián cảm)

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Từ khóa:
Hố lường tử, hiệu ứng Ettingshausen, hiệu ứng từ-nhiet-dien, phương trình dòng lượng tử, phonon âm gián cảm.

Tóm tắt
Biểu thức của hệ số Ettingshausen trong hố lường tử với hố thể parabol khi có sóng điện tử được thu nhận trên cơ sở phương trình dòng lượng tử cho ham phần bối của điện tử trong trường hợp tán xạ điện tử - phonon âm gián cảm. Các kết quả giải tích đã chỉ ra sự phụ thuộc phức tạp của hệ số Ettingshausen vào nhiệt độ, từ trường, các đại lượng đặc trưng của sóng điện tử và số lượng từ m đặc trưng cho phonon gián cảm. Những sự phụ thuộc này được hiện thị rõ nét trong kết quả tính toán số cho hố lường tử GaAs/GaAsAl. Đặc biệt, khi cho m tiến về không, ta thu được kết quả của hiệu ứng từ-nhiet-dien tương ứng với trường hợp phonon không gián cảm trong hố lường tử cùng loại.