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Finite-dimensional states and entanglement generation

Doan Quoc Khoa[a], Luong Thi Tu Oanh[b], Chu Van Lanh[b], Nguyen Thi Dung[c], Nguyen Van Hoa[c],*

[a]*Quang Tri Teacher Training College*
[b]*Vinh University*
[c]*Hong Duc University*
[*]*Email: khoa_dqspqt@yahoo.com*

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Abstract

We studied a system that has two nonlinear oscillators with different frequencies. These oscillators are coupled via a nonlinear interaction. With the aim of excite the system, we used two external coherent fields. It follows from numerical simulation that evolution of the system which is likely to be a combination of Fock states. Therefore, the deliberated system behaves as so-called nonlinear quantum scissor. However, evolution of the system generates Bell-like states in some times with exact high probability. The system creates a truncation of optical states, which leads to obtain two-qubit states due to the nonlinear properties of oscillators and their interaction.

1. Introduction

There are numerous published works in quantum information, which concentrates studies of scientists for some decades and several of them are in the form of monographs [1, 2]. One of the central problems in this field is looking mechanisms for create entanglement states in suitable physical systems that can create a set of n-photon states, named ordinarily as Kerr-like nonlinear systems. These systems often include two nonlinear oscillators, which coupler with each other in a linear or nonlinear method. A common method to obtain special phenomena in nonlinear optics is use laser to influence optical systems. Therefore, one is able to find the quantum scissors, which can generate a finite number of states from the infinite-dimension states in the Hilbert space. Building the linear quantum scissors [3, 4] or nonlinear quantum scissors [5, 6], depend on the used optical parts and the style of their interaction. Lately, several special systems, in which the coupler between two nonlinear oscillators are able to modeled by Werner-like states [7] or delta function [8] are considered. The generation of maximally entangled states is most important result of these discussions. In these works, several enjoyable phenomena have been found out as sudden death and birth of entanglement states [9] or an exciting field phase effect [10]. Here, we suggest a model that consist of a Kerr-like nonlinear coupler pumped in two modes in which two oscillators nonlinearly coupler with each other and restrict ourselves to the case without damping and compare our results with that obtained previously.

2. The model

The model of the nonlinear coupler here consist of two nonlinear oscillators that are specific to Kerr nonlinearities χ_a and χ_b with the field modes *a* and *b*, respectively. The nonlinear oscillators are nonlinearity coupler with each other and pumped by two linear external coherent fields. Then, the Hamiltonian depicting this system has the form as

$$\hat{H} = \hat{H}_N + \hat{H}_{\text{int}} + \hat{H}_{ext} \qquad (1)$$

in which

$$\hat{H}_N = \frac{\chi_a}{2}\left(\hat{a}^+\right)^2 \hat{a}^2 + \frac{\chi_b}{2}\left(\hat{b}^+\right)^2 \hat{b}^2, \tag{2}$$

$$\hat{H}_{\text{int}} = \mu\left(\hat{a}^+\right)^2 \hat{b}^2 + \mu^* \hat{a}^2 \left(\hat{b}^+\right)^2, \tag{3}$$

$$\hat{H}_{ext} = \sigma\hat{a}^+ + \sigma^*\hat{a} + \theta\hat{b}^+ + \theta^*\hat{b}, \tag{4}$$

where component $\hat{H}_N$ is a term of Hamiltonian which describes nonlinear oscillators in two modes *a* and *b*, $\hat{H}_{\text{int}}$ describes coupler between the modes while $\hat{H}_{ext}$ depicts interaction of the modes with linear external coherent fields. $\hat{a}$ and $\hat{b}$ are boson annihilation operators, whereas $\hat{a}^+$ and $\hat{b}^+$ are boson creation operators corresponding to two modes *a* and *b*, respectively. We use complex parameters σ and θ to depict coupling strength between the modes *a* and *b* with external coherent fields, respectively. The parameter μ is the coupler strength between two oscillators in the model.

The evolution of our system is depicted by wave function of the time-dependent that is defined by the Schrödinger equation in interaction picture has the following form

$$i\frac{d}{dt}\left|\psi(t)\right\rangle = \hat{H}\left|\psi(t)\right\rangle, \tag{5}$$

in which time-dependent wave function $\left|\psi(t)\right\rangle$ depicting evolution of the system with complex probability amplitudes $c_{pq}(t)$ as

$$\left|\psi(t)\right\rangle = \sum_{p,q=0}^{\infty} c_{pq}(t)\left|p\right\rangle_a \left|q\right\rangle_b. \tag{6}$$

One can be cut (6) to the time-dependent wave function which is depicted with only some n-photon states of an infinite number of photons by use the formalism of nonlinear quantum scissors as in [5]. Then the time-dependent wave function of system can be represented only in a set of four states $\left|2\right\rangle_a\left|0\right\rangle_b$, $\left|0\right\rangle_a\left|2\right\rangle_b$, $\left|2\right\rangle_a\left|1\right\rangle_b$ and $\left|1\right\rangle_a\left|2\right\rangle_b$ with the following form

$$\begin{aligned}\left|\psi(t)\right\rangle_{cut} &= c_{02}(t)\left|0\right\rangle_a\left|2\right\rangle_b + c_{12}(t)\left|1\right\rangle_a\left|2\right\rangle_b + \\ & c_{21}(t)\left|2\right\rangle_a\left|1\right\rangle_b + c_{20}(t)\left|2\right\rangle_a\left|0\right\rangle_b\end{aligned} \tag{7}$$

For general case, we put $\sigma = \kappa\mu$. Besides that, we suppose that the linear external fields have the same strength ($\sigma = \theta$) with σ and θ are real numbers and the initial state of the system has zero photon in mode *a* and two photons in mode *b* in the cavity, namely $c_{20}(0) = c_{12}(0) = c_{21}(0) = 0$ and $c_{02}(0) = 1$. Then the solution of complex amplitudes $c_{20}(t)$, $c_{12}(t)$, $c_{21}(t)$ and $c_{02}(t)$ have the following form

$$\begin{aligned}
c_{20}(t) &= -\frac{i}{2\zeta}\left(\Omega_1 \sin(\Omega_1\tau) - \Omega_2 \sin(\Omega_2\tau)\right),\\
c_{12}(t) &= \frac{\zeta^2 - 1}{2\zeta}\left(\cos(\Omega_1\tau) - \cos(\Omega_2\tau)\right),\\
c_{21}(t) &= -\frac{i}{2}\left(\frac{\Omega_1}{\omega_1^2 - 1}\sin(\Omega_2\tau) + \frac{\Omega_2}{\Omega_2^2 - 1}\sin(\Omega_1\tau)\right),\\
c_{02}(t) &= \frac{\zeta + 1}{2\zeta}\left(\cos(\Omega_1\tau) - \cos(\Omega_2\tau)\right),
\end{aligned} \tag{8}$$

where

$$\Omega_1 = \sqrt{\zeta^2 + \zeta + 1}, \quad \Omega_2 = \sqrt{\zeta^2 - \zeta + 1},$$

$$\zeta = \sqrt{\kappa^2 + 1}, \quad \kappa = \frac{\sigma}{\mu}, \quad \tau = \mu t. \tag{9}$$

3. Generation of maximally entangled states

We will be here dedicated to depict the states that are generated by our model. A significant feature of the model is its ability to create maximally entangled states (Bell-like states) $\left|\psi(t)\right\rangle_{cut}$. To see that, we depict time-evolution of entanglement in terms of the von Neumann entropy, which is discussed in [1, 8, 9]. From the formula (6), the full density matrix $\rho_{ab} = \left|\psi(t)\right\rangle_{cutcut}\left\langle\psi(t)\right|$, indications the time-evolution of the system. The partial trace of ρ_{ab} with respect to the mode b has the following form

$$\begin{aligned}\rho_b = Tr_a\rho_{ab} &= \left|c_{20}\right|^2 \left|0\right\rangle_{bb}\left\langle 0\right| \\ &+ c_{20}c_{21}^* \left|0\right\rangle_{bb}\left\langle 1\right| + c_{21}c_{20}^* \left|1\right\rangle_{bb}\left\langle 0\right| \\ &+ \left|c_{21}\right|^2 \left|1\right\rangle_{bb}\left\langle 1\right| + \left(\left|c_{02}\right|^2 + \left|c_{12}\right|^2\right)\left|2\right\rangle_{bb}\left\langle 2\right|.\end{aligned} \tag{10}$$

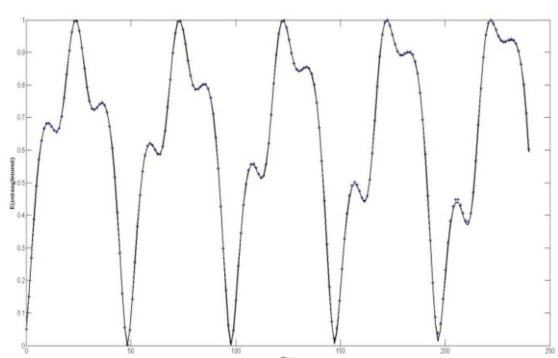

Figure 1: Time-evolution of entropy of entanglement E of the generated $\left|\psi(t)\right\rangle$ (dots) and the truncated state

$\left|\psi(t)\right\rangle_{cut}$ *(solid curve). The coupling strengths* $\sigma = \pi/20$ *rad/s,* $\mu = \sigma/4$ *rad/s and the nonlinearities* $\chi_a = \chi_b = 20$ *rad/s. Time unit is scaled in 1/χ.*

Therefore, the entropy von Neumann has the following form

$$\begin{aligned} E &= -Tr\rho_a \log_2 \rho_a = -Tr\rho_b \log_2 \rho_b \\ &= -\lambda_1 \log_2 \lambda_1 - \lambda_2 \log_2 \lambda_2, \end{aligned} \tag{11}$$

in which λ_1 and λ_2 are eigenvalues of ρ_b.

The entropy of entanglement is shown in the Fig. 1. We can show that the highest entropy of entanglement of the state $\left|\psi(t)\right\rangle_{cut}$ is almost equal to unit for some moments of time. From there, we can see that the system behaves as a nonlinear quantum scissor that can create maximally entangled states, namely the key maximum and minimum values of entanglement divide each other with a period $T \approx 50$ (1/χ unit). This exhibition allows us to believe that the maximally entangled states may be created in some times.

We expect that, our system can create maximally entangled states. Two of the Bell-like states $\left|B_1\right\rangle$ and $\left|B_2\right\rangle$ that have the form as

$$\left|B_1\right\rangle = \frac{1}{\sqrt{2}}\left(\left|2\right\rangle_a\left|0\right\rangle_b + i\left|0\right\rangle_a\left|2\right\rangle_b\right), \tag{12}$$

$$\left|B_2\right\rangle = \frac{1}{\sqrt{2}}\left(\left|2\right\rangle_a\left|0\right\rangle_b - i\left|0\right\rangle_a\left|2\right\rangle_b\right). \tag{13}$$

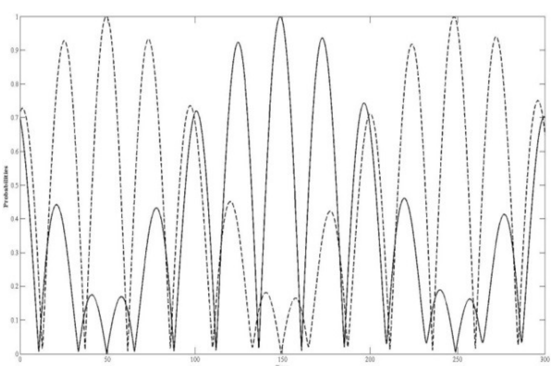

Figure 2: The probabilities corresponding to the Bell-like states $\left|B_1\right\rangle$ *(solid curve),* $\left|B_2\right\rangle$ *(dashed curve). The parameters are the same in the figure 1.*

We can generate of Bell-like states $\left|B_i\right\rangle$ by calculating the probabilities of output state in Bell-like states $\left|\left\langle\psi(t)\right|B_i\right\rangle\right|$. The probabilities corresponding to the output state in Bell-like states $\left|B_1\right\rangle$ and $\left|B_2\right\rangle$ are plotted in figure 2. We show that in this figure, the maximum probabilities of these states are equal to unit at some moments of time. These results are greater than that are shown in [10]. Then, we trust that the output state of our system can be in Bell-like states $\left|B_i\right\rangle$ (i = 3,...,8) with the following form

$$\left|B_3\right\rangle = \frac{1}{\sqrt{2}}\left(\left|2\right\rangle_a\left|0\right\rangle_b + \left|0\right\rangle_a\left|2\right\rangle_b\right), \tag{14}$$

$$\left|B_4\right\rangle = \frac{1}{\sqrt{2}}\left(\left|2\right\rangle_a\left|0\right\rangle_b - \left|0\right\rangle_a\left|2\right\rangle_b\right), \tag{15}$$

$$\left|B_5\right\rangle = \frac{1}{\sqrt{2}}\left(\left|1\right\rangle_a\left|2\right\rangle_b + \left|2\right\rangle_a\left|0\right\rangle_b\right), \tag{16}$$

$$\left|B_6\right\rangle = \frac{1}{\sqrt{2}}\left(\left|1\right\rangle_a\left|2\right\rangle_b - \left|2\right\rangle_a\left|0\right\rangle_b\right), \tag{17}$$

$$\left|B_7\right\rangle = \frac{1}{\sqrt{2}}\left(\left|2\right\rangle_a\left|0\right\rangle_b + i\left|1\right\rangle_a\left|2\right\rangle_b\right), \tag{18}$$

$$\left|B_8\right\rangle = \frac{1}{\sqrt{2}}\left(\left|2\right\rangle_a\left|0\right\rangle_b - i\left|1\right\rangle_a\left|2\right\rangle_b\right). \tag{19}$$

The probabilities corresponding to states $\left|B_3\right\rangle$ and $\left|B_4\right\rangle$ are shown in figure 3, $\left|B_5\right\rangle$ and $\left|B_6\right\rangle$ are shown in figure 4 and $\left|B_7\right\rangle$ and $\left|B_8\right\rangle$ are plotted in figure 5. In these figures we can see that time-evolution of the probabilities corresponding to states $\left|B_i\right\rangle$ (i = 3,...,8) approximate to 0.7 and they have the same forms in pairs ($\left|B_3\right\rangle$, $\left|B_4\right\rangle$), ($\left|B_5\right\rangle$, $\left|B_6\right\rangle$) and ($\left|B_7\right\rangle$, $\left|B_8\right\rangle$). In addition, these pairs are only different from the other by phase part. It means that there occur entangled states in time-evolution of this system but they are not really Bell-states.

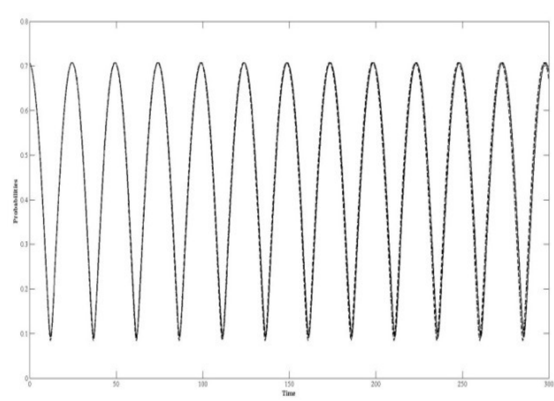

Figure 3: The probabilities corresponding to the Bell-like states $|B_3\rangle$ (solid curve), $|B_4\rangle$(dashed curve). The parameters are the same in the previous figures.

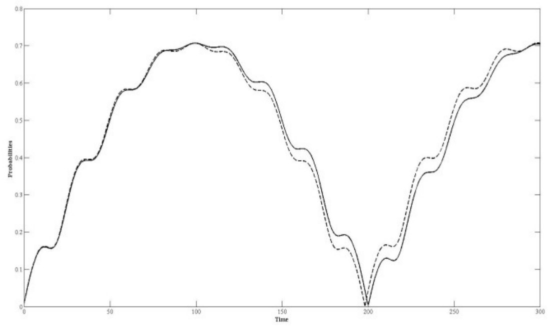

Figure 4: The probabilities corresponding to the Bell-like states $|B_5\rangle$ (solid curve), $|B_6\rangle$(dashed curve). The parameters are the same in the previous figures.

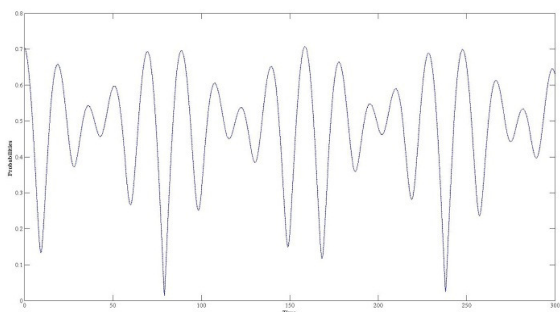

Figure 5: The probabilities corresponding to the Bell-like states $|B_7\rangle$ (solid curve), $|B_8\rangle$(dashed curve). The parameters are the same in the previous figures.

4. Conclusions

In this work, a system containing two nonlinear oscillators that is nonlinearity coupled with each other and pumped in two modes have been discussed. By using the nonlinear scissors formalism, we can be expressed our system in a set of four states $|2\rangle_a|0\rangle_b$, $|0\rangle_a|2\rangle_b$, $|2\rangle_a|1\rangle_b$ and $|1\rangle_a|2\rangle_b$. The maximally entangled values of the system are almost equal to 1[ebit] in a greater number of moments of time than the results in [6]. Thus, this system can create the maximally entangled states with closely the same results in analytical calculation and numerical simulation. Moreover, we shown that the entanglement and evolution of the system are dependent on phase difference between two linear pumped fields. Thus, we can expect that our system can be applied both as a source to obtain maximally entangled states and as a component of more complex systems used in quantum information.

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Các trạng thái hữu hạn chiều và sự tạo trạng thái đan rối

Đoàn Quốc Khoa[a], *Lương Thị Tú Oanh* [b], *Chu Văn Lanh* [b], *Nguyễn Thị Dung* [c], *Nguyễn Văn Hóa*[c],

[a]*Trường Cao đẳng Sư phạm Quảng Trị*
[b]*Trường Đại học Vinh*
[c]*Trường Đại học Hồng Đức*

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Tóm tắt

Chúng tôi nghiên cứu hệ gồm hai dao động tử phi tuyến với tần số khác nhau. Các dao động tử này tương tác phi tuyến với nhau. Để kích thích hệ, chúng tôi sử dụng hai trường kết hợp ngoài. Từ mô phỏng số chỉ ra rằng, sự tiến triển của hệ giống như tổ hợp của các trạng thái Fock. Vì vậy, hệ hoạt động như kéo lượng tử phi tuyến. Tuy nhiên, sự tiến triển của hệ tạo ra các trạng thái kiểu Bell trong một số lần với xác suất rất cao. Hệ tạo ra sự cắt cụt các trạng thái quang học, dẫn đến việc tìm được các trạng thái hai qubit do tính chất phi tuyến của các dao động tử và tương tác của chúng.