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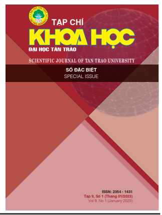

INFLATION AND AXION DARK MATTER IN THE 3-3-1 MODEL

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Abstract:

We consider a renormalizable theory, which successfully explains the number of Standard Model (SM) fermion families and whose non-SM scalar sector includes an axion dark matter as well as a field responsible for cosmological inflation. In such theory, the axion gets its mass via radiative corrections at one-loop level mediated by virtual top quark, right handed Majorana neutrinos and SM gauge bosons. Its mass is obtained in the range 4keV÷40keV, consistent with the one predicted by XENON1T experiment, when the right handed Majorana neutrino mass is varied from 100GeV up to 350GeV, thus implying that the light active neutrino masses are generated from a low scale type I seesaw mechanism. Furthermore, the theory under consideration can also successfully accommodates the XENON1T excess provided that the PQ symmetry is spontaneously broken at the 10^{10} GeV scale.

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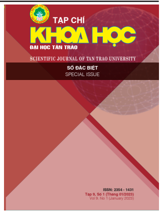

LẠM PHÁT VŨ TRỤ CHẤT TỐI AXION TRONG MÔ HÌNH 3-3-1

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Từ khóa:

Vật chất tối axion,

Cơ chế seesaw I

Đối xứng PQ

Tóm tắt

Chúng tôi nghiên cứu một lý thuyết tái chuẩn hóa thành công trong việc giải thích số thế hệ fermion trong Mô hình chuẩn. Trong lý thuyết này, phần vô hướng không thuộc Mô hình chuẩn là phần có chứa đồng thời vật chất tối axion và một trường tương ứng với lạm phát vũ trụ. Với lý thuyết như vậy, hạt axion nhận khối lượng thông qua bức xạ bổ đính một vòng với quark đỉnh là hạt ảo đóng vai trò trung gian, các hạt neutrino Majorana xoay phải và các boson chuẩn của Mô hình chuẩn. Khối lượng của axion được tính toán nằm trong khoảng 4 đến 40 keV. Điều này phù hợp với kết quả của thí nghiệm XENON1T khi khối lượng của neutrino Majorana xoay phải nhận giá trị trong khoảng từ 100 đến 350 GeV và ngụ ý rằng khối lượng của neutrino hoạt tính được sinh ra từ cơ chế cầu bập bênh loại I ở thang năng lượng thấp. Hơn nữa, lý thuyết được nghiên cứu cũng có thể thành công trong việc giải thích được sự phá vỡ đối xứng PQ một cách tự phát ở thang năng lượng 1010 GeV khi sử dụng kết quả thu được từ thí nghiệm XENON1T.

I. INTRODUCTION

Currently, an axion is a very attractive subject in Particle Physics in both theoretical and experimental aspects [1,2], thus providing a motivation to consider extensions of the Standard Model that include this particle in its field content. The axion is a CP-odd scalar field which arises in the solution of the strong-CP problem, and originally it is always massless particle; and in order to generate a mass for the axion, one can consider the implementation of radiative corrections as shown in [3] or gravitational effects [4]. It is interesting to note that nowadays the axion is widely considered as a candidate of dark matter (DM)[5]. The dark matter candidate is existed only in some beyond standard models. Among the SM extensions, the models based on the $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{U}(1)_X$ gauge symmetries (called 3-3-1 models, for short) [6-13] have several very interesting features, some of them being, the natural explanation of the number of SM fermion families, the electric charge quantization and the solution of the strong CP problem from the PQ symmetry [14], which is automatically fulfilled in the 3-3-1 models. In one of the 3-3-1 models, there exist both interesting features, namely the axion dark matter candidate and inflaton for Early Universe [15,16]. In the above mentioned papers, the axion gets mass only from gravitational contribution.

It is worth mentioning that the CP-odd sector of the 3-3-1 model with axion has been considered in [15-17].

However, the rotation matrix that diagonalizes the squared mass matrix for the CP odd neutral scalar fields given in [15], which was obtained by Mathematica is not unitary, and thus such mixing matrix cannot be used for further studies such coupling constants, collider phenomenology, etc. The aim of this work is to reconsider the CP-odd scalar sector of the aforementioned model with the inclusion of radiative corrections mediated by virtual top quark, right handed Majorana neutrinos and SM gauge bosons, which in turn generate the mass of the axion. Such radiative corrections to the axion mass where not considered in [15]. Our paper is organized as follows: in Section II we present brief review of the model. Section III is devoted to discrete and PQ symmetries needed for the axion existence. The Higgs potential and the resulting physical scalar spectrum is discussed in Section IV. In Section V, we present radiative correction for the axion mass. Finally, we state our conclusions in Section VI.

II. REVIEW OF THE MODEL

The model under consideration is based on $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{U}(1)_X \times Z_{11} \times Z_2$ and has the following fermion content:

$$\psi_{aL} = (\nu_a, l_a, (\nu_R^c)^a)_L^T \sim \left(1,3,-\tfrac{1}{3}\right),\ l_{aR} \sim (1,1,-1),$$
$$N_{aR} \sim (1,1,0), Q_{3L} = (u_3, d_3, U)_L^T \sim \left(\tfrac{3,3,1}{3}\right),$$
$$Q_{\alpha L} = (d_\alpha, -u_\alpha, D_\alpha)_L^T \sim (3,3^*,0),$$
$$u_{aR}, U_R \sim \left(\tfrac{3,1,2}{3}\right),\ d_{aR}, D_{\alpha R} \sim \left(3,1,-\tfrac{1}{3}\right),$$

where $\alpha = 1,2$ and $a = \{3,\alpha\}$ are family indices. The U and D are exotic quarks with ordinary electric charges, whereas N_{aR} are right-handed neutrinos.

The scalar sector of the model is composed of three $SU(3)_L$ scalar triplets and one $SU(3)_L$ singlet scalar field. They have the following transformations under the $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{U}(1)_X$ symmetry:

$$\chi^T = (\chi_1^0, \chi_2^-, \chi_3^0) \sim \left(1,3,-\frac{1}{3}\right), \eta^T = (\eta_1^0, \eta_2^-, \eta_3^0)$$
$$\rho^T = (\rho_1^+, \rho_2^0, \rho_3^+) \sim \left(1,3,\frac{2}{3}\right),\ \phi \sim (1,1,0).$$

To generate masses for gauge bosons and fermions, the scalar fields should acquire vacuum expectation values (VEVs). These fields can be expanded around the minimum as follows

$$\chi = \begin{pmatrix} \frac{1}{\sqrt{2}}(R_\chi^1 + iI_\chi^1) \\ \chi^- \\ \frac{1}{\sqrt{2}}(v_\chi + R_\chi^3 + iI_\chi^3) \end{pmatrix}, \eta = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_\eta + R_\eta^1 + iI_\eta^1) \\ \eta^- \\ \frac{1}{\sqrt{2}}(R_\eta^3 + iI_\eta^3) \end{pmatrix},$$
$$\rho = \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(v_\rho + R_\rho + iI_\rho) \\ \rho_3^+ \end{pmatrix}, \phi = \frac{1}{\sqrt{2}}(v_\phi + R_\phi + iI_\phi).$$

Note that since ϕ carries non-zero PQ charge (as shown below), it has to be complex as shown in (3). The VEV v_χ is responsible for the first stage of gauge symmetry breaking, whereas v_η, v_ρ trigger the second stage of electroweak symmetry breaking.

III. DISCRETE AND PECCEI-QUINN SYMMETRIES

In order to keep intact the physics results of the Ref.[15] the Lagrangian of the model must be invariant by the discrete symmetries $Z_{11} \times Z_2$ which are summarised in Table I. Here we have used a notation $\omega_k \equiv e^{i2\pi\frac{k}{11}}, k = 0, \pm 1 \cdots \pm 5$.

Table I: $\mathrm{SU(3)_C \times SU(3)_L \times U(1)_X \times Z_{11} \times Z_2}$ charge assignments of the particle content of the model. Here $\mathbf{a = 1,2,3}$ and $\boldsymbol{\alpha = 1,2}$.

	$Q_{\alpha L}$	Q_{3L}	u_{aR}	d_{aR}	U_{3R}	$D_{\alpha R}$	ψ_{aL}	l_{aR}	N_{aR}	η	χ	ρ	ϕ
$SU(3)_C$	**3**	**3**	**3**	**3**	**3**	**3**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$SU(3)_L$	$\mathbf{\overline{3}}$	**3**	**1**	**1**	**1**	**1**	**3**	**1**	**1**	**3**	**3**	**3**	**1**
$U(1)_X$	0	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	-1	0	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	0
Z_{11}	ω_4^{-1}	ω_0	ω_5	ω_2	ω_3	ω_4	ω_1	ω_3	ω_5^{-1}	ω_5^{-1}	ω_3^{-1}	ω_2^{-1}	ω_1^{-1}
Z_2	1	1	-1	-1	-1	-1	1	-1	1	1	-1	-1	1

From Table I, ones recognize that under Z_2 symmetry, the following fields are odd

$$\eta, \rho, u_R, d_{\alpha R}, e_{\alpha R}, N_R) \rightarrow -(\eta, \rho, u_R, d_{\alpha R}, e_{\alpha R}, N_R).$$

Let us discuss about the PQ symmetry. These discrete symmetries yield the following Yukawa couplings

$$-\mathcal{L}^Y = y_1 \bar{Q}_{3L} U_{3R} \chi + \sum_{\alpha,\beta=1}^{2} (y_2)_{\alpha\beta} \bar{Q}_{\alpha L} D_{\beta R} \chi^* + \sum_{a=1}^{3} (y_3)_{3a} \bar{Q}_{3L} u_{aR} \eta + \sum_{\alpha=1}^{2} \sum_{a=1}^{3} (y_4)_{\alpha a} \bar{Q}_{\alpha L} d_{aR} \eta^* + \sum_{a=1}^{3} (y_5)_{3a} \bar{Q}_{3L} d_{aR} \rho + \sum_{\alpha=1}^{2} \sum_{a=1}^{3} (y_6)_{\alpha a} \bar{Q}_{\alpha L} u_{aR} \rho^* + \sum_{a=1}^{3} \sum_{b=1}^{3} g_{ab} \bar{\psi}_{aL} e_{bR} \rho + \sum_{a=1}^{3} \sum_{b=1}^{3} (y_\nu^D)_{ab} \bar{\psi}_{aL} \eta N_{bR} + \sum_{a=1}^{3} \sum_{b=1}^{3} (y_N)_{ab} \phi \bar{N}_{aR}^C N_{bR} + \text{H.c.}.$$

where unlike [15], the neutrino Yukawa term $y_{ab}^{(\rho)} (\bar{\psi}_{aL})(\psi_{bL})(\rho^*)$ is not present since that such term is not invariant under the Z_2 symmetry. It is worth mentioning that the Yukawa interactions given above do not allow for terms which interchange $\chi \leftrightarrow \eta$, since they do not respect the $Z_{11} \times Z_2$ symmetry.

Assuming fermions of opposite chiralities have opposite PQ charges and $X_d = X_D = 1$ we summarise PQ charges of fermions in Table II.

Table II: PQ charges of fermions in the model

	u_{aL}	d_{aL}	U_L	D_L	l_a	l_{aR}	ν_{aL}	ν_{aR}	N_{aR}
X_{PQ}	-1	1	1	1	1	1	1	-1	1

It is now clear that the entire Lagrangian of the model is $U_{PQ}(1)$ invariant, providing a natural solution to the strong-CP problem.

The following remarks are in order

i) The discrete symmetry Z_N can naturally be accommodated when it has enough number of fields in its spectrum.

ii) If the $Z_{11} \times Z_2$ symmetry is imposed, then the PQ symmetry automatically appears in the model. Therefore, the CP problem can be solved by the dynamical properties of the axion field, which is a physical scalar field belonging to the CP odd neutral scalar sector.

iii) From the last two terms of Eq. (5), it follows that the tiny masses for the light active neutrinos are generated from a type I seesaw mechanism mediated by right handed Majorana neutrinos, thus implying that the resulting light active neutrino mass matrix has the form:

$$M_\nu = M_\nu^D M_N^{-1} (M_\nu^D)^T, M_\nu^D = y_\nu^D \frac{v_\eta}{\sqrt{2}}, M_N = y_N \frac{v_\phi}{\sqrt{2}}.$$

IV. HIGGS POTENTIAL

The model scalar potential takes the form:

$$\begin{aligned} V = {} & \mu_\phi^2 \phi^* \phi + \mu_\chi^2 \chi^\dagger \chi + \mu_\rho^2 \rho^\dagger \rho + \mu_\eta^2 \eta^\dagger \eta + \lambda_1 (\chi^\dagger \chi)^2 + \lambda_2 (\eta^\dagger \eta)^2 \\ & + \lambda_3 (\rho^\dagger \rho)^2 + \lambda_4 (\chi^\dagger \chi)(\eta^\dagger \eta) + \lambda_5 (\chi^\dagger \chi)(\rho^\dagger \rho) + \lambda_6 (\eta^\dagger \eta)(\rho^\dagger \rho) \\ & + \lambda_7 (\chi^\dagger \eta)(\eta^\dagger \chi) + \lambda_8 (\chi^\dagger \rho)(\rho^\dagger \chi) + \lambda_9 (\eta^\dagger \rho)(\rho^\dagger \eta) \\ & + \lambda_{10} (\phi^* \phi)^2 + \lambda_{11} (\phi^* \phi)(\chi^\dagger \chi) + \lambda_{12} (\phi^* \phi)(\rho^\dagger \rho) \\ & + \lambda_{13} (\phi^* \phi)(\eta^\dagger \eta) + \left(\lambda_\phi \epsilon^{ijk} \eta_i \rho_j \chi_k \phi + H.c. \right) \end{aligned}$$

The VEV v_ϕ is responsible for PQ symmetry breaking resulting in existence of invisible axion due to very high scale around $10^{10} - 10^{11}$GeV. Then $\mathrm{SU}(3)_L \times \mathrm{U}(1)_X$ breaks to the SM group by v_χ and two others v_ρ, v_η are needed for the usual $\mathrm{U}(1)_Q$ symmetry. Hence $v_\phi \gg v_\chi \gg v_\rho, v_\eta$. The constraint conditions of such scalar potential were analyzed in Ref. [15].

From an analysis of the scalar potential, we find that the physical CP odd neutral scalar mass eigenstates are:

$$\begin{pmatrix} G_{Z'} \\ A_5 \\ G_Z \\ a \end{pmatrix} = \begin{pmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ \sin\theta_3\cos\theta_4\cos\theta_\phi & \cos\theta_3\cos\theta_4\cos\theta_\phi & -\sin\theta_4\cos\theta_\phi & \sin\theta_\phi \\ -\sin\theta_3\sin\theta_4 & \sin\theta_4\cos\theta_3 & \cos\theta_4 & 0 \\ -\sin\theta_3\sin\theta_\phi\cos\theta_4 & -\sin\theta_\phi\cos\theta_3\cos\theta_4 & -\sin\theta_4\sin\theta_\phi & \cos\theta_\phi \end{pmatrix} \begin{pmatrix} I_\chi^3 \\ I_\eta^1 \\ I_\rho \\ I_\phi \end{pmatrix}$$

where the mixing angles in the CP odd scalar sector take the form:

$$\begin{aligned} \tan\alpha &= \frac{v_\eta}{v_\rho},\ \tan\theta_3 = \frac{v_\eta}{v_\chi}, \\ \tan\theta_\phi &= \frac{v_\rho v_\eta}{v_\phi\sqrt{v_\rho^2 + v_\eta^2}} \approx \frac{v_\rho}{v_\phi}.\ \tan\theta_4 \approx \tan\alpha. \end{aligned}$$

The mixing angles in the CP odd scalar sector depend on the ratio of v_η to v_ρ, v_χ and v_ϕ.

By rotating the scalar fields to the physical basis, considering the limit $v_\chi \gg v_\eta, v_\rho$ and a scenario close to the decoupling limit $v_\eta \gg v_\rho$, it follows that the scalar fields of the model under consideration can be expressed as follows [18,19] :

$$\chi \simeq \begin{pmatrix} G_{X^0} \\ G_{Y^-} \\ \frac{1}{\sqrt{2}}\left(v_\chi + H_\chi + iG_{Z'}\right) \end{pmatrix}, \eta \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}\left(v_\eta + h + iG_Z\right) \\ G_{W^-} \\ \varphi^0 \end{pmatrix}$$

$$\rho \simeq \begin{pmatrix} H_1^+ \\ \frac{1}{\sqrt{2}}\left(v_\rho + h_5 + iA_5\right) \\ H_2^+ \end{pmatrix}, \phi \simeq \frac{1}{\sqrt{2}}\left(v_\phi + \Phi + ia\right).$$

The scalar spectrum of the model has the following features:

1) In the charged scalar sector, one has four singly charged fields: two of them are massless being the Goldstones bosons eaten by the longitudinal components of the bilepton gauge bosons $W^\pm$and $Y^\pm$, whereas the physical charged scalar fields have masses at the TeV scale.

2) In the CP-odd scalar sector, there are six fields: two massless Goldstones eaten by the longitudinal components of the Z and Z' gauge bosons and one massless is part of Goldstones boson associated to the longitudinal components of the neutral bilepton G_{X^0} gauge boson. One massive CP-odd field has the same mass as another in CP-even sector. Hence the above pair is a complex bilepton scalar field denoted by φ^0. Another field associated with the singlet ϕ is identified to the axion a. Originally, this axion is massless, but it becomes massive thanks to radiative orrections at one loop level. Therefore, at tree level we have only one massive CP-odd scalar field A_5.

3) In the CP-even scalar sector, there are six fields. One massless field is part of G_{X^0}, another massive is associated to φ^0. One heavy field associated with the imaginary part of the singlet ϕ is identified with the inflaton Φ and one SM-like Higgs boson h with mass 126GeV. Besides that, in order to trigger inflation, the inflaton Φ should acquire a very large vacuum expectation value of around 10^{11}GeV. For a detailed study of how the field Φ drives inflation in 3-3-1 models, see for instance Ref [16, 20]. The remaining two fields are one heavy with mass at TeV scale and another with mass at EW scale.

4) After including the axion field $a(x)$, the total Lagrangian as follows

$$\begin{aligned} \mathcal{L}_{\text{Total}} &= \mathcal{L}_{SM} + \left(\bar{\theta} + \frac{a(x)}{f_a}\right)\frac{g_s^2}{32\pi^2} G^{a\mu\nu}\tilde{G}^a{}_{\mu\nu} \\ &+ \text{kinetic} + \text{interactions} \end{aligned}$$

where the axion decay constant $f_a = \frac{v_\phi}{\sqrt{2}}$, is related to the magnitude of the VEV that breaks the $U(1)_{PQ}$ symmetry. Hence, the CP violating term $G\tilde{G}$ is proportional to $\left(\bar{\theta} + \frac{a(x)}{f_a}\right)$, when the axion field is redefined, $a(x) \to a(x) - \langle a(x)\rangle$, thus implying that the CP violating term $G\tilde{G}$ is no longer present in the Lagrangian, so that the Strong CP problem is solved.

5) Another important point is that the invisible axion of the model has a small component of I_χ^3 which couples only to exotic quarks at tree level via the following Yukawa interaction

$$\begin{aligned} \mathcal{L}_{(aq'q')} &= -\frac{i}{\sqrt{2}}\left(\sin\theta_3 \sin\theta_\phi \cos\theta_4\right) \\ &[h^U \bar{U}_L U_R - h^D_{\alpha\beta}\bar{D}_{\alpha L} D_{\beta R}]a + H.c \end{aligned}$$

6) From Eq. (8) we find

$$I_\rho = -\sin\theta_4\cos\theta_\phi A_5 + \cos\theta_4 G_Z - \sin\theta_4\sin\theta_\phi a.$$

Hence from Eq. (5) we obtain that the electronpositron-axion interaction has the form:

$$\begin{aligned} L_{ae} &= -i\frac{\tilde{g}_{11}}{\sqrt{2}}\sin\theta_\phi \sin\theta_4 a\bar{e}\gamma_5 e \\ &= -i\frac{\sqrt{2}m_e}{v_\rho}\sin\theta_\phi \sin\theta_4 a\bar{e}\gamma_5 e \equiv -ig_{ae}a\bar{e}\gamma_5 e \end{aligned}$$

where $m_e = 0.488$MeV is the electron mass at the M_Z scale. Considering, $v_\rho = v_\eta = \frac{v}{\sqrt{2}} \approx 174$GeV, which corresponds to $\theta_4 \approx \frac{\pi}{2}, v_\phi \approx 1.95 \times 10^{10}$GeV, we get the following value for the electron-positron-axion coupling:

$$g_{ae} = 2.5 \times 10^{-14},$$

which is consistent with its corresponding experimental value $g_{ae} = 2.5 \times 10^{-14}$ arising the best fit data arising from the XENON1T excess [25].

7) From Eqs. (5) and (8), we conclude that the interactions among axion and fermions are, as expected, proportional to the masses of the latter.

V. THE AXION MASS

At tree level, the axion is massless. In order to generate the axion mass, there are two methods: using the gravitational effect as done in [16] or relying on radiative corrections. In this paper, the latter method is used. Before proceeding with the calculation of the loop diagrams, it is worth mentioning that for a successful reheating after inflation [16] and for avoiding troubles with divergences in higher order interactions of the Higgs in electroweak phase transition [21], the scalar couplings between axion and scalar fields such as: $\lambda_\phi, \lambda_{11}, \lambda_{12}, \lambda_{13}$ must be very small around $10^{-10} - 10^{-6}$. Therefore, in the self-energy correction of the axion, the following limits are applied: i) $\sin\theta_3 \simeq \frac{v_\eta}{v_\chi} \to 0$. Within these considerations, the dominant contribution to the axion mass at one-loop level consists of the following diagrams: ii) Loop with fermions and gauge bosons in the internal lines. However, in the concerning to the fermionic contributions, we only consider the leading ones which correspond to the loops with the top quark and right-handed neutrinos $N_{aR} (a = 1,2,3)$. iii) Tadpole diagrams with the W, Z, Z', X and Y gauge bosons. Hence the axion mass is given by

$$
\begin{aligned}
-\frac{m_a^2}{2} &= 12\frac{1}{v_\eta^2}\sin^2\theta_\phi\cos^2\theta_4 m_t^4\left[1-\ln\left(\frac{m_t^2}{m_Z^2}\right)\right]+\frac{12}{v_\phi^2}m_N^4\left[1-\ln\left(\frac{m_N^2}{m_Z^2}\right)\right]+3\sum_{V=Z,W,Z',X,Y} g_{aaVV}m_V^2\left[1-\ln\left(\frac{m_V^2}{m_Z^2}\right)\right]\\
&+\sum_{V=Z,W,Z',X,Y,H=h,h_5,.} 4g_{aVH}^2\left[-m_H^2\left(\ln\frac{m_H^2}{m_Z^2}-1\right)+m_V^2\left(-1+\frac{m_V^2+m_H^2}{m_V^2-m_H^2}\right)\ln\frac{m_V^2}{m_H^2}\right]
\end{aligned}
$$

where the terms in (14) are the contributions due to t, N_R and gauge bosons, respectively. In (14), the necessary quartic couplings are given by

$$
\begin{aligned}
g_{aaW^+W^-} &= \frac{1}{2}g^2\sin^2(\theta_\phi)\left(\sin^2(\theta_4)+\cos^2(\theta_3)\cos^2(\theta_4)\right),\\
g_{aaZZ} &= \frac{g^2\sin^2(\theta_\phi)\left(\sin^2(\theta_4)+\cos^2(\theta_3)\cos^2(\theta_4)\right)}{c_W^2},\\
g_{aaZ'Z'} &= \frac{TC}{8s_W^4-14s_W^2+6}, g_{aaX^*X} = \frac{1}{2}g^2\cos^2(\theta_4)\sin^2(\theta_\phi)\\
g_{aaY^+Y^-} &= \frac{1}{2}g^2\sin^2(\theta_\phi)\left(\sin^2(\theta_4)+\sin^2(\theta_3)\cos^2(\theta_4)\right)
\end{aligned}
$$

where

$$
\begin{aligned}
TC &= 2g^2\sin^2(\theta_\phi)\left(\sin^2(\theta_4)+\frac{1}{2}\left(5-3\cos(2\theta_3)\right)\cos^2(\theta_4)\right.\\
&\left.+2\cos^2(\theta_4)s_W^2\left(\cos(2\theta_3)+2s_W^2-3\right)\right)
\end{aligned}
$$

The triple couplings are determined as follows

i) For neutral gauge bosons

$$L_{aVR} = g_{aVR}V_\mu(R\,\partial^\mu a - a\,\partial^\mu R),$$

where V stands for the neutral gauge bosons Z and Z', and R are the physical CP-even Higgs bosons. The calculation yields

$$
\begin{aligned}
g_{aZh_5} &= -\frac{g}{4c_W}\sin\theta_\phi\sin\theta_4,\ g_{aZh} = \frac{g}{4c_W}\sin\theta_\phi\cos\theta_3\cos\theta_4\\
g_{aZ'h_5} &= \frac{g}{4c_W\sqrt{3-4s_W^2}}\sin\theta_\phi\sin\theta_4,\\
g_{aZ'h} &= \frac{g}{4c_W\sqrt{3-4s_W^2}}\sin\theta_\phi\cos\theta_3\cos\theta_4\cos(2\theta_W),\\
g_{aZ'H_\chi} &= -2\frac{g}{4c_W\sqrt{3-4s_W^2}}\sin\theta_\phi\sin\theta_3\cos\theta_4,
\end{aligned}
$$

where h is the SM-like Higgs boson.

ii) For the charged gauge bosons, one has

$$L_{aVC} = g_{aVC} V_\mu^+ (C^- \partial^\mu a - a\,\partial^\mu C^-)$$
$$+H.c., V = W, Y; C = H_1, H_2$$
$$L_{aXH_\chi} = g_{aXH_\chi} X_\mu^0 (H_\chi \partial^\mu a - a\,\partial^\mu H_\chi)$$

with

$$g_{aWH_1} = \frac{g}{2\sqrt{2}} \sin\theta_\phi \cos\theta_3 \cos\theta_4, g_{aYH_2} = \frac{g}{2\sqrt{2}} \sin\theta_\phi \sin\theta_4,$$
$$g_{aXH_\chi} = \frac{g}{2\sqrt{2}} \sin\theta_\phi \sin\theta_3.$$

In this model, the masses of the non SM gauge bosons satisfy the relation, hence in our calculation, we assume

$$m_Y^2 \simeq m_X^2 \simeq \frac{g^2}{4} v_\chi^2 \simeq \frac{(3-4s_W^2)}{4} m_{Z'}^2,$$

Using data in [27,30], we take $m_{Z'} \geq 4$TeV. The numerical evaluation shows that the correlation between the axion and the Majorana neutrino masses, for $0.5\text{TeV} \leqslant m_{H_5} \leqslant 10\text{TeV}, 4\text{TeV} \leqslant m_{H_\chi} \leqslant 10\text{TeV}, 0.5\text{TeV} \leqslant m_{H_1^\pm} \leqslant 1\text{TeV}, 2\text{TeV} \leqslant m_{H_2^\pm} \leqslant 10\ \text{TeV}, 4\text{TeV} \leqslant m_{Z'} \leqslant 8\text{TeV}$ and $10 \leqslant \tan\alpha \leqslant 50$. Furthermore, we have set $v_\phi = 1.95 \times 10^{10}$GeV. Numerical analysis show that the axion gets mass in the range $4\text{keV} \div 40\text{keV}$, when the right handed Majorana neutrino mass is varied from 100GeV up to 350GeV. Notice that the XENON1T experiment predicts a range for the axion mass [23] consistent with our results. For the BBN bound where $m_a \in (20\text{keV} \div 1\text{MeV})$[24], the mass of N_R is in the range $270\text{GeV} \div 1375\text{GeV}$

VI. CONCLUSIONS

The model under consideration contains three important features: Dark Matter axion, inflation and low scale type I seesaw mechanism to generate the light active neutrino masses. In addition, from the analysis of the model scalar potential we find that in the CP-odd neutral scalar sector, there are six fields: one massive CP-odd field A_4, two massless Goldstones eaten by the longitudinal components of the Z and Z' and one massless which is part of Goldstones boson associated to the longitudinal component of the neutral bilepton G_{X^0} gauge boson. One massive CP-odd field has the same mass of another one in the CP-even sector. Hence the above pair is a complex bilepton scalar field denoted by φ^0. Finally, another field associated to the neutral scalar singlet ϕ is identified with axion a which becomes massive due to radiative corrections at one-loop level, mediated by virtual top quark, right handed Majorana neutrinos and SM gauge bosons. Its mass is obtained in the range 4 keV ÷ 40keV, coincided with the one predicted by XENON1T experiment, when the right handed Majorana neutrino mass is varied from 100GeV up to 350GeV, thus implying that the light active neutrino masses are generated from a low scale type I seesaw mechanism as in [26]. In the model under consideration, the couplings between axion and fermion are proportional to the fermion masses. Such axion is also very useful for successfully accommodating the XENON1T excess provided that the PQ symmetry is spontaneously broken at the 10^{10}GeV scale.

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