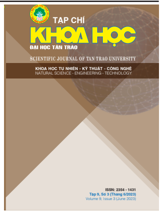
TẠP CHÍ KHOA HỌC ĐẠI HỌC TÂN TRÀO

ISSN: 2354 - 1431

http://tckh.daihoctantrao.edu.vn/



GLOBAL ATTRACTORS FOR A NONCLASSICAL DIFFUSION EQUATION LACKING INSTANTANEOUS DAMPING WITH MEMORY

Dang Thi Phuong Thanh
Hung Vuong University, Vietnam
**Email address: thanhdp83@gmail.com*
http://doi.org/10.51453/2354-1431/2023/970

Article info

Recieved:
06/3/2023
Revised:
10/4/2023
Accepted:
15/5/2023

Keywords:
Nonclassical diffusion; lacking instantaneous damping; memory; global attractor; semigroup

Abstract:

In this paper, we consider a periodic boundary value problem for a nonclassical diffusion equation lacking instantaneous damping with hereditary memory

$$u_t - \Delta u_t - \int_0^\infty \kappa(s)\Delta u(t-s)ds + k_0|u|^{p-1}u = g.$$

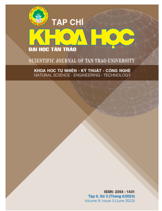
The main characteristics of the model is that the equation does not contain a term of the form $-\Delta u$, which contributes to an instantaneous damping. We use the ω-limit compactness of the solution semigroup $\{S(t)\}_{t\geq 0}$ to get the existence of a global attractor.

Vol 9. No 3_May 2023

TẠP CHÍ KHOA HỌC ĐẠI HỌC TÂN TRÀO

ISSN: 2354 - 1431

http://tckh.daihoctantrao.edu.vn/



TẬP HÚT TOÀN CỤC CỦA PHƯƠNG TRÌNH KHUẾCH TÁN KHÔNG CỔ ĐIỂN CHỨA NHỚ KHUYẾT SỐ HẠNG TẮT DẦN TỨC THỜI

Đặng Thị Phương Thanh

Trường Đại học Hùng Vương

**Email address: thanhdp83@gmail.com*

http://doi.org/10.51453/2354-1431/2023/970

Thông tin bài viết

Ngày nhận bài:
06/3/2023
Ngày sửa bài:
10/4/2023
Ngày duyệt đăng:
15/5/2023

Từ khóa:
Khuếch tán không cổ điển; khuyết số hạng tắt dần tức thời; nhớ; tập hút toàn cục; nửa nhóm

Tóm tắt:

Trong bài báo này, chúng tôi xét bài toán giá trị biên cho lớp phương trình khuếch tán không cổ điển có nhớ và khuyết số hạng tắt dần tức thời

$$u_t - \Delta u_t - \int_0^\infty \kappa(s)\Delta u(t-s)ds + k_0|u|^{p-1}u = g.$$

Đặc điểm chính của mô hình là phương trình không chứa số hạng có dạng $-\Delta u$, điều này góp phần tạo ra sự tắt dần tức thời. Chúng tôi sử dụng tính chất ω - giới hạn compact của nửa nhóm $\{S(t)\}_{t\geq 0}$ để chứng minh sự tồn tại của tập hút toàn cục.

1 INTRODUCTION

The study of the asymptotic behavior of dynamical systems arising from mechanics and physics is a capital issue, as it is essential, for practical applications, to be able to get understood and even predict the long-time behavior of the solutions of such systems. One way to attack the problem for a dissipative dynamical system is to consider its global attractor and some related issues.

The main goal of this paper is to discuss the long-time behavior of the solutions for the following partial differential equation

$$u_t - \Delta u_t - \int_0^\infty \kappa(s)\Delta u(t-s)ds + k_0|u|^{p-1}u = g(x) \tag{1.1}$$

with the damping coefficient $k_0 > 0$, $2 \leq p \leq 5$, and the initial data

$$u(x,0) = u_0(x), \quad x \in \mathbb{R}^n,\ t \leq 0, \tag{1.2}$$

$$u(x,t) = u(x+Le_i,t), x \in \mathbb{R}^n,\ t \in \mathbb{R},\ i = 1,2\ldots,n, \tag{1.3}$$

where $e_i = \{0,\ldots,0,1,0,\ldots,0\}$ and L is a positive constant.

The nonclassical diffusion equation was introduced by E.C. Aifantis [1] as a model to describe physical phenomena, such as non-Newtonian flows, soil mechanics and heat conduction theory (see, e.g., [1]). The usual nonclassical diffusion equation has form

$$u_t - \Delta u_t - \Delta u + f(u) = g. \tag{1.4}$$

In the last two decades, the existence and long-time behavior of solutions has extensively been studied by many authors, for both in bounded domain case (see [7]) and in unbouded domain case (see [8]), and even in non-cylindrical domains (see [5]).

In the case of the nonclassical diffusion equation containing viscoelasticity of the conductive medium, that is to say, we add a fading memory term to this equation,

$$u_t - \Delta u_t - \Delta u - \int_0^\infty \kappa(s)\Delta u(t-s)ds + f(u) = g. \tag{1.5}$$

The conduction of energy is not only affected by present external forces but also by historic external forces. This leads to the speed of energy dissipation for equation (1.5) which is faster than for the equation (1.4). The existence and long-time behavior of solutions to nonclassical diffusion equations with memory has been investigated for both autonomous case (see [9]) and non-autonomous case (see [6]) where the memory term satisfies the conditions

$$\kappa(s) = \int_s^\infty \mu(r)dr, \quad \mu'(s) + \delta\mu(s) \le 0; \tag{1.6}$$

or only requires conditions much weaker than (1.6)

$$\kappa(s) \le \Theta\mu(s), \tag{1.7}$$

which can be equivalently expressed in the form $\mu(\sigma+s) \le Me^{-\delta\sigma}\mu(s)$.

In particular, V. Pata *et al.* [2] considered the nonclassical diffusion equation with memory lacking instantaneous damping

$$u_t - \Delta u_t - \int_0^\infty \kappa(s)\Delta u(t-s)ds + f(u) = g. \tag{1.8}$$

Under assumption on the memory kernel as in (1.6), and the nonlinearity f fulfills the growth restriction

$$|f(u) - f(v)| \le k_f|u-v|(1+|u|^4+|v|^4)$$
$$\text{for some } k_f > 0,$$

they proved the existence of global attractors and its regularity. Besides, they pointed out that the whole dissipation is contributed by the convolution term only. Then, the weaker condition (1.7) no longer suffices to ensure the decay of the energy.

In this paper, based on the idea in [2], we prove the existence of the global attractor $\mathcal{A} \in \dot{H}^2_{\text{per}}(\Omega) \times L^2_\mu(\mathbb{R}^+, \dot{H}^2_{\text{per}}(\Omega))$ for the nonclassical diffusion with memory lacking instantaneous damping. We use the periodic boundary conditions in $\mathbb{R}^n$ space instead of the Dirichlet conditions on the bounded domain Ω, which are discussed widely, and we get the existence of bouned absorbing sets in the function spaces $\dot{H}^1_{\text{per}}(\Omega) \times L^2_\mu(\mathbb{R}^+, \dot{H}^1_{\text{per}}(\Omega))$ and $\dot{H}^2_{\text{per}}(\Omega) \times L^2_\mu(\mathbb{R}^+, \dot{H}^2_{\text{per}}(\Omega))$, respectively, then we use the ω-limit compactness of the solution semigroup $\{S(t)\}_{t\ge0}$ to get the existence of global attractor.

Now, denote $\Omega = \prod_{i=1}^n (0,L)$ and the spaces

$$\dot{L}^2_{\text{per}}(\Omega) = \{u \in L^2(\Omega) : \int_\Omega u dx = 0, u \text{ is periodic in } x \text{ with period } L\},$$

$$\dot{H}^s_{\text{per}}(\Omega) = \{u \in H^s(\Omega) : \int_\Omega u dx = 0, u \text{ is periodic in } x \text{ with period } L\},$$

$(\cdot,\cdot)$ and $\|\cdot\|$ are inner product and norm of $\dot{L}^2_{\text{per}}(\Omega)$ along with $(\cdot,\cdot)_s$ and $\|\cdot\|_s$ are inner product and norm of $\dot{H}^s_{\text{per}}(\Omega)$.

To study the problem (1.1), we assume that the external force g and the memory kernel κ satisfy the following conditions

(H1) *The time-independent external force* $g \in L^2(\Omega)$.

(H2) *The memory kernel* κ *is a nonnegative summable function (we take it of unitary mass) of the form*

$$\kappa(s) = \int_s^\infty \mu(r)dr,$$

where $\mu \in L^1(\mathbb{R}^+)$ *is a decreasing (hence nonnegative) piecewise absolutely continuous function such that*

$$\mu'(s) + \delta\mu(s) \le 0, \tag{1.9}$$

for some $\delta > 0$ *and almost every* $s > 0$. *In particular, these assumptions imply that*

$$\kappa(0) = \int_0^\infty \mu(s)ds < \infty, \text{ and } \mu(0) < \infty,$$

namely, μ *can be continuously extended to the origin. To avoid the presence of unnecessary constants, from now on we assume* $\kappa(0) = 1$ *which can be always obtained by rescaling the memory kernel.*

We mention here, that μ *can be taken to be unbounded in the origin; also, it is allowed to exhibit downward jumps, even countably many.*

To this aim, following C.M. Dafermos [3], we consider a new variable reflecting history of (1.1) which is introduced, that is to be,

$$\eta^t(x,s) = \eta(x,t,s) = \int_0^s u(x,t-r)dr, \quad s \geq 0,$$

then we can check that

$$\partial_t \eta^t(x,s) = u(x,t) - \partial_s \eta^t(x,s), \ s \geq 0.$$

Since $\mu(s) = -\kappa'(s)$, problem (1.1) can be transformed into the following system

$$\begin{cases} u_t - \Delta u_t - \int_0^\infty \mu(s)\Delta\eta^t(x,s)ds + k_0|u|^{p-1}u \\ \qquad = g(x), \ x \in \mathbb{R}^n, t \in \mathbb{R}, \\ \partial_t \eta^t(x,s) = -\partial_s \eta^t(x,s) + u(x,t), \\ \qquad x \in \mathbb{R}^n, t > \tau, s \geq 0, \\ u(x,t) = u(x+L,t), \ x \in \mathbb{R}^n, t \leq 0, \\ \eta^t(x,s) = \eta^t(x+L,s), \ x \in \mathbb{R}^n, s \in \mathbb{R}^+, t \leq 0, \\ u(x,0) = u_0(x), \ x \in \mathbb{R}^n, \\ \eta^0(x,s) = \eta_0(x,s) = \int_0^s u_0(x,-\tau)d\tau, \\ \qquad x \in \mathbb{R}^n, s \in \mathbb{R}^+. \end{cases} \tag{1.10}$$

Denote

$$z(t) = (u(t), \eta^t), \text{ and } z_0 = (u_0, \eta_0).$$

We now define the *history spaces* $L^2_\mu(\mathbb{R}^+, \dot{H}^r_{\text{per}}(\Omega))$, which is the Hilbert space of functions $\varphi\colon \mathbb{R}^+ \to H^r_{\text{per}}(\Omega)$ endowed with the inner product

$$\langle \varphi_1, \varphi_2 \rangle_{r,\mu} = \int_0^\infty \mu(s) \langle \varphi_1(s), \varphi_2(s) \rangle_r \, ds,$$

and let $\|\cdot\|_{r,\mu}$ denote the corresponding norm.

We now introduce the following Hilbert spaces

$$\mathcal{H}_i = \dot{H}^i_{\text{per}}(\Omega) \times L^2_\mu(\mathbb{R}^+, \dot{H}^i_{\text{per}}(\Omega)), i = 1, 2.$$

We first get the existence and uniqueness of local solution to the problem (1.10) as follows.

Theorem 1.1. *Assume that hypotheses* **(H1)-(H2)** *hold. Then for any $z_0 = (u_0, \eta_0) \in \mathcal{H}_1$, then the problem* (1.10) *has a unique solution $z = (u, \eta^t)$ on the interval* $[0,T]$ *satisfying*

$$z \in C([0,T]; \mathcal{H}_1) \quad \text{for all } T > 0.$$

Furthermore, if $z_0 \in \mathcal{H}_2$, then the problem (1.10) *has a unique solution $z = (u, \eta^t)$, it satisfies that*

$$z \in C([0,T]; \mathcal{H}_2) \quad \text{for all } T > 0.$$

The proof of the result follows by an application of a Galerkin scheme, based on the energy estimates of the forthcoming Lemma 2.3 and Lemma 2.4.

The paper is organized as follows. In Section 1, we introduce the notation along with some definitions and give some assumptions on the forcing term g as well as the memory kernel $\kappa(\cdot)$ (or $\mu(\cdot)$). In Section 2, we investigate the existence of global attractors $\mathcal{A}$.

2 THE EXISTENCE OF A GLOBAL ATTRACTOR IN $\mathcal{H}_2$

The aim of this section is to prove the following theorem.

Theorem 2.1. *Assume that conditions* **(H1)-(H2)** *hold. The semigroup $\{S(t)\}_{t\geq 0}$ generated by* (1.10) *has a global attractor $\mathcal{A}$ in $\mathcal{H}_2$.*

In order to obtain the existence of global attractor, we first prove existence of bounded absorbing sets in the spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, then we show that the semigroup $\{S(t)\}_{t\geq 0}$ is the ω-limit compact.

The proof of existence of the absorbing set exploits in a crucial way the following technical lemma.

Lemma 2.2. *Assume that (u, η^t) is a sufficiently regular solution to* (1.10). *Then, for $a > 0$, the functional*

$$\Lambda_i(t) = -\langle u(t), \eta^t \rangle_{i,\mu}, \quad i = 1,\, 2,$$

fulfills the differential inequality

$$\begin{aligned} &\frac{d}{dt}\Lambda_i(t) + \frac{1}{2}\|u(t)\|_i^2 \\ &\quad \leq \frac{a}{4}\|u_t(t)\|_i^2 + \frac{1}{a}\|\eta^t(t)\|_{i,\mu}^2 \\ &\quad + \frac{\mu(0)}{2}\int_0^\infty -\mu'(s)\|\eta^t\|_i^2 ds. \end{aligned}$$

Besides, we have the control

$$|\Lambda_i(t)| \leq \frac{1}{2}E_i,$$

where $E_i = \|u\|_i^2 + \|\eta^t(t)\|_{i,\mu}^2$ for $i = 1,\, 2$.

The proof is very similar to that in [2, Lemma 3.3], so we omit it.

Now, we prove the existence of the absobing sets in $\mathcal{H}_1$ and $\mathcal{H}_2$.

2.1 Existence of a Bounded Absorbing Set in $\mathcal{H}_1$

Lemma 2.3. *Let* **(H1)-(H2)** *hold. Then there exists a bounded absorbing set in* $\mathcal{H}_1$ *for the semigroup* $\{S(t)\}_{t\geq 0}$.

Chứng minh. Taking the inner product in $\dot{L}^2_{\rm per}(\Omega)$ of (1.10) with $u(t)$, we have

$$\frac{1}{2}\frac{d}{dt}\left(\|u\|^2+\|\nabla u\|^2\right)+k_0\|u\|^{p+1}_{\dot{L}^{p+1}_{\rm per}} + \int_0^\infty \mu(s)\langle\nabla\eta^t(s),\nabla u\rangle ds=(g,u).$$

Noting that $\eta^t_t=-\eta^t_s+u$, we have

$$\begin{aligned}&\int_0^\infty \mu(s)\int_\Omega \nabla\eta^t\nabla u dxds\\ &=\int_0^\infty \mu(s)\int_\Omega \nabla\eta^t\nabla\eta^t_t dxds\\ &+\int_0^\infty \mu(s)\int_\Omega \nabla\eta^t\nabla\eta^t_s dxds\\ &=\frac{1}{2}\frac{d}{dt}\|\eta^t\|^2_{1,\mu}-\int_0^\infty \mu'(s)\|\eta^t\|^2_1 ds. \qquad (2.1)\end{aligned}$$

Combining (2.1) and (2.1), we get

$$\frac{d}{dt}E_1+2\int_0^\infty -\mu'(s)\|\eta^t\|^2_1 ds+2k_0\|u\|^{p+1}_{\dot{L}^{p+1}_{\rm per}}=2(g,u). \qquad (2.2)$$

We now multiply the first equation of (1.10) by u_t in $\dot{L}^2_{\rm per}(\Omega)$, we get

$$\begin{aligned}&\|u_t\|^2+\|\nabla u_t\|^2+\frac{2k_0}{p+1}\frac{d}{dt}\|u(t)\|^{p+1}_{\dot{L}^{p+1}_{\rm per}}\\ &=\langle g,u_t\rangle-\int_0^\infty \mu(s)\langle\nabla\eta^t(s),\nabla u_t\rangle ds\\ &\leq\frac{1}{2}(\|u_t\|^2+\|\nabla u_t\|^2)+\frac{1}{2}\|\eta^t\|^2_{1,\mu}+\frac{1}{2}\|g\|^2.\end{aligned}$$

Thus,

$$\frac{4k_0}{p+1}\frac{d}{dt}\|u(t)\|^{p+1}_{\dot{L}^{p+1}_{\rm per}}+(\|u_t\|^2+\|\nabla u_t\|^2)\leq\|\eta^t\|^2_{1,\mu}+\|g\|^2. \qquad (2.3)$$

Next, for $\nu,a\in(0,1)$, we define the function

$$\Phi_1(t)=E_1+a^2\left(\frac{4k_0}{p+1}\|u(t)\|^{p+1}_{\dot{L}^{p+1}_{\rm per}}\right)+\nu a\Lambda_1(t),$$

where $\Lambda_1(t)$ is defined in Lemma 2.2. Thanks to $p\in[2,5]$, it follows

$$\frac{1}{2}E_1(t)\leq\Phi_1(t)\leq 2E_1(t)+C\|u(t)\|^{p+1}_{\dot{L}^{p+1}_{\rm per}}.$$

Combining (2.2) and (2.3) and using Lemma 2.2, we obtain

$$\begin{aligned}&\frac{d}{dt}\Phi_1(t)+\frac{\nu a}{4}\|u\|^2_1+\left(a^2-\frac{\nu a^2}{4}\right)(\|u_t\|^2+\|u_t\|^2_1)\\ &+\left(2-\frac{\nu a\mu(0)}{2}\right)\int_0^\infty -\mu'(s)\|\eta^t\|^2_1 ds+k_0\|u\|^{p+1}_{\dot{L}^{p+1}_{\rm per}}\\ &\leq(a^2+\nu)\|\eta^t\|^2_{1,\mu}+C\|g\|^2, \qquad (2.4)\end{aligned}$$

where $2(g,u)\leq\frac{\nu a}{4}\|u\|^2_1+C\|g\|^2$.
Using the condition (1.9), and choosing $\nu,a>0$ are small enough, we have

$$\frac{d}{dt}\Phi_1(t)+\frac{\nu a}{4}\|u\|^2_1+\frac{\delta}{2}\|\eta^t\|^2_{1,\mu}+k_0\|u\|^{p+1}_{\dot{L}^{p+1}_{\rm per}}\leq C\|g\|^2,$$

then there exists a constant $\gamma>0$ such that

$$\frac{d}{dt}\Phi_1+\gamma\Phi_1\leq C\|g\|^2.$$

Using the Gronwall inequality, we obtain

$$\Phi_1(t)\leq\Phi_1(0)e^{-\gamma t}+C\|g\|^2,$$

thus,

$$E_1(t)\leq\rho_1 \text{ or } \|z(t)\|^2_{\mathcal{H}_1}\leq\rho_1,$$

for all $z_0\in B$ and for all $t\geq T_B$, where B is an arbitrary bounded subset of $\mathcal{H}_1$. This completes the proof. □

2.2 Existence of a Bounded Absorbing Set in $\mathcal{H}_2$

Lemma 2.4. *Let* **(H1)-(H2)** *hold. Then there exists a bounded absorbing set in* $\mathcal{H}_2$ *for the semigroup* $\{S(t)\}_{t\geq 0}$.

Chứng minh. Multiplying the first equation of (1.10) by $-\Delta u(t)$ in $\dot{L}^2_{\rm per}(\Omega)$, we have

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}E_2(t)+k_0\int_\Omega |u|^{p-1}|\nabla u|^2dx\\ &+\frac{k_0(p-1)}{4}\int_\Omega |u|^{p-3}|\nabla|u|^2|^2dx-\int_0^\infty \mu'(s)\|\eta^t(s)\|^2_2 ds\\ &=(g,-\Delta u),\end{aligned}$$

where $E_2=\|u\|^2_1+\|u\|^2_2+\|\eta^t\|^2_{2,\mu}$. At last, we define the function

$$\Phi_2(t)=E_2(t)+\nu a\Lambda_2(t),$$

where $\Lambda_2(t)$ is defined in Lemma 2.2 and $\frac{1}{2}E_2(t)\leq\Phi_2(t)\leq 2E_2(t)$.

Using the condition (1.9), we can see that $-\mu'(s)\geq\delta\mu(s)$, then $\Phi_2(t)$ satisfies the differential inequality

$$\begin{aligned}&\frac{d}{dt}\Phi_2(t)+\left(2\delta-\frac{\nu a\delta\mu(0)}{2}-\nu\right)\|\eta^t\|^2_{2,\mu}+\frac{\nu a}{4}\|u\|^2_2\\ &\leq\frac{\nu a^2}{4}\|u_t\|^2_2+C\|g\|^2, \qquad (2.5)\end{aligned}$$

where

$$(g, -\Delta u) \leq \frac{\nu a}{4}\|u\|_2^2 + C\|g\|^2.$$

We now multiply the first equation of (1.10) by $-\Delta u_t$, we get

$$\begin{aligned}\|u_t\|_1^2 + \|u_t\|_2^2 &= \langle g, -\Delta u_t\rangle + k_0\int_\Omega |u|^{p-1}u \cdot \Delta u_t dx \\ &\quad - \int_0^\infty \mu(s)\langle \Delta\eta^t(s), \Delta u_t\rangle ds \\ &\leq \frac{1}{2}\|u_t\|_2^2 + 2\|\eta^t\|_{2,\mu}^2 + k_0^2\|u\|_2^2 + 2\|g\|^2,\end{aligned}$$

where

$$\begin{aligned}\langle g, -\Delta u_t\rangle - \int_0^\infty \mu(s)\langle \Delta\eta^t(s), \Delta u_t\rangle ds \\ \leq \frac{1}{4}\|u_t\|_2^2 + 2\|\eta^t\|_{2,\mu}^2 + 2\|g\|^2,\end{aligned}$$

and

$$k_0\int_\Omega |u|^{p-1}u \cdot \Delta u_t dx \leq \frac{1}{4}\|u_t\|_2^2 + k^2\|u\|_2^2.$$

Thus,

$$\begin{aligned}&a^2(\|u_t\|_1^2 + \|u_t\|_2^2) \\ &\quad \leq a^2\left(4\|\eta^t\|_{2,\mu}^2 + 2k_0^2\|u\|_2^2 + 4\|g\|^2\right). \qquad (2.6)\end{aligned}$$

Summation of (2.5) and (2.6), we obtain

$$\begin{aligned}&\frac{d}{dt}\Phi_2(t) + \left(2\delta - \frac{\nu a\delta\mu(0)}{2} - \nu - 4a^2\right)\|\eta^t\|_{2,\mu}^2 \\ &+ \left(\frac{\nu a}{4} - 2a^2k_0^2\right)\|u\|_2^2 + a^2\left(1 - \frac{\nu}{4}\right)(\|u_t\|_1^2 + \|u_t\|_2^2) \\ &\leq C\|g\|^2.\end{aligned}$$

Choosing $a, \nu > 0$ are small enough then there exists a constant $\gamma > 0$ such that

$$\frac{d}{dt}\Phi_2(t) + \gamma_2\Phi_2(t) \leq C\|g\|^2.$$

Applying Gronwall inequality, we get

$$\Phi_2(t) \leq \Phi_2(0)e^{-\gamma_2 t} + C\|g\|^2. \qquad (2.7)$$

Hence there exists $\rho_2 > 0$ such that

$$E_2(t) \leq \rho_2 \quad \text{or} \quad \|z(t)\|_{\mathcal{H}_2}^2 \leq \rho_2, \qquad (2.8)$$

for all $z_0 \in B$ and for all $t \geq T_B$, where B is an arbitrary bounded subset of $\mathcal{H}_2$. This completes the proof. □

2.3 The Proof of Theorem 2.1

We now give a detailed proof of the Theorem 2.1.

Chứng minh. Firstly, we prove that the semigroup $\{S(t)\}_{t\geq 0}$ is ω-limit compact. We have a set $\{w_k\}$ with eigenvalue λ_k,

$$\begin{aligned}&-\Delta w_k(x) = \lambda_k w_k(x), \\ &w_k(x + Le_i) = w_k(x),\ i = 1, 2, \ldots, n, \\ &\int_\Omega w_k(x)dx = 0,\end{aligned}$$

λ_k is an orthonormal basis for $\mathcal{H}_2$, they also form an orthogonal basis for $\mathcal{H}_1$. Denote $H_m = \text{span}\{w_1, w_2, \ldots, w_m\}$, and let P_m be the orthogonal projector on H_m and I be the identity. Then for any $(u, \eta^t) \in \mathcal{H}_2$, u has a unique decomposition

$$(u, \eta^t) = (u_1, \eta_1^t) + (u_2, \eta_2^t),$$

where

$$\begin{aligned}&(u_1, \eta_1^t) = (P_m u, P_m\eta^t) \quad \text{and} \\ &(u_2, \eta_2^t) = ((I - P_m)u, (I - P_m)\eta^t).\end{aligned}$$

On the other hand, for any $u \in \dot{H}^2_{\text{per}}(\Omega)$ we can see that $f(u) := |u|^{p-1}u \in \dot{H}^1_{\text{per}}(\Omega)$ and $f : \dot{H}^2_{\text{per}}(\Omega) \to \dot{H}^1_{\text{per}}(\Omega)$ is continuously compact, then for all $\varepsilon > 0$, there exists m such that

$$\|(I - P_m)g\| \leq \frac{\varepsilon}{4}, \qquad (2.9)$$

$$\|(I - P_m)f(u)\|_1 \leq \frac{\varepsilon}{4} \quad \text{for all } u \in B_2(0, \rho_2). \qquad (2.10)$$

Multiplying (1.10) by $-\Delta u_2$ and using the similar method described in subsection 2.2, we have

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\left(\|u_2\|_1^2 + \|u_2\|_2^2 + \|\eta_2^t\|_{2,\mu}^2\right) + k_0\int_\Omega f(u)\Delta u_2 dx \\ &- \int_0^\infty \mu'(s)\|\eta_2^t(s)\|_2^2 ds = (g, -\Delta u_2), \qquad (2.11)\end{aligned}$$

where $f(u) = |u|^{p-1}u$.

Inequality (2.9) implies that

$$\begin{aligned}(g, -\Delta u_2) &= (P_m g + (I - P_m)g, -\Delta u_2) \\ &= (g_1 + g_2, -\Delta u_2) \\ &\leq \|g_2\|\|u_2\|_2 \leq \frac{\varepsilon}{4}\|u_2\|_2 \leq \frac{\nu a}{4}\|u_2\|_2^2 + \frac{\varepsilon^2}{16\nu a}, \qquad (2.12)\end{aligned}$$

and from (2.10), we obtain

$$\begin{aligned}&k_0\left|\int_\Omega f(u)\Delta u_2 dx\right| \\ &= k_0\left|\int_\Omega \nabla f(u)\nabla u_2 dx\right| \\ &\leq k_0\|f(u)\|_1\|\nabla u_2\| \\ &\leq \frac{k_0\varepsilon}{4}\|\nabla u_2\| \\ &\leq \frac{k_0^2\varepsilon^2}{16\lambda_2} + \frac{\lambda_2}{4}\|\nabla u_2\|^2, \qquad (2.13)\end{aligned}$$

where λ_2 satisfies that $\|\Delta u_2\|^2 \geq \lambda_2 \|\nabla u_2\|^2$ for all $u_2 \in D(-\Delta)$.

Combining (2.11)-(2.13), we deduce that

$$\frac{d}{dt}E_3(t) - 2\int_0^\infty \mu'(s)\|\eta_2^t(s)\|_2^2 ds \leq \frac{\nu a}{4}\|u_2\|_2^2 + \frac{k_0^2\varepsilon^2}{8\lambda_2} + \frac{\varepsilon^2}{8\nu a}.$$

where $E_3(t) = \|u_2\|_1^2 + \|u_2\|_2^2 + \|\eta_2^t\|_{2,\mu}^2$.

Next, we introduction the function

$$\Phi_3(t) = E_3(t) + \nu a \Lambda_2$$

where $\Lambda_2(t)$ is defined in Lemma 2.2 (with u_2 in place of u) and

$$\frac{1}{2}E_3(t) \leq \Phi_3(t) \leq 2E_3(t).$$

Using the condition (1.9), then $\Phi_3(t)$ satisfies the differential inequality

$$\frac{d}{dt}\Phi_3(t) + \left(2\delta - \frac{\nu a\delta\mu(0)}{2} - \nu\right)\|\eta_2^t\|_{2,\mu}^2 + \frac{\nu a}{4}\|u_2\|_2^2 \leq \frac{\nu a^2}{4}\|\partial_t u_2\|_2^2 + \frac{k_0^2\varepsilon^2}{8\lambda_2} + \frac{\varepsilon^2}{8\nu a}. \quad (2.14)$$

We now multiply the first equation of (1.10) by $-\Delta u_t$, then using similar arguments as in (2.6), one can deduce that

$$a^2(\|\partial_t u_2\|_1^2 + \|\partial_t u_2\|_2^2) \leq a^2\left(4\|\eta_2^t\|_{2,\mu}^2 + 2k_0^2\|u_2\|_2^2 + \varepsilon^2\right). \quad (2.15)$$

Summation of (2.14) and (2.15), we obtain

$$\frac{d}{dt}\Phi_2(t) + \left(2\delta - \frac{\nu a\delta\mu(0)}{2} - \nu - 4a^2\right)\|\eta^t\|_{2,\mu}^2 + \left(\frac{\nu a}{4} - 2a^2k_0^2\right)\|u\|_2^2 + a^2\left(1 - \frac{\nu}{4}\right)(\|u_t\|_1^2 + \|u_t\|_2^2) \leq \frac{k_0^2\varepsilon^2}{8\lambda_2} + \frac{\varepsilon^2}{8\nu a} + \varepsilon^2 = C\varepsilon^2.$$

We now can choose ν, $a > 0$ small enough, then there exists a constant $\gamma > 0$ such that

$$\frac{d}{dt}\Phi_3(t) + \gamma_3\Phi_3(t) \leq C\varepsilon^2 \quad \text{for all } t \geq T_B.$$

By the Gronwall inequality, we have

$$\Phi_3(t) \leq \Phi_3(T_B)e^{-\gamma_3(t-T_B)} + C\varepsilon^2 \quad \text{for all } t \geq T_B.$$

Thus,

$$\|u_2\|_1^2 + \|u_2\|_2^2 + \|\eta_2^t\|_{2,\mu}^2 \leq C\left(\|u_2(T_B)\|_1^2 + \|u_2(T_B)\|_2^2 + \|\eta_2^{T_B}\|_{2,\mu}^2\right)e^{-\gamma_3(t-T_B)} + C\varepsilon^2 \leq \rho_2^2 e^{-\gamma_3(t-T_B)} + C_0\varepsilon^2 \quad \text{for all } t \geq T_B,$$

where ρ_2 and T_B are defined in (2.8) in Lemma 2.4. Choosing T^* large enough such that $T^* - T_B \geq \frac{2}{\gamma_3} ln\left(\frac{\rho_2}{\varepsilon}\right)$, then

$$\|u_2\|_1^2 + \|u_2\|_2^2 + \|\eta_2^t\|_{2,\mu}^2 \leq (1 + C_0)\varepsilon^2 = C_1\varepsilon^2 \quad \text{for all } t \geq T^*.$$

Therefore $\{S(t)\}_{t\geq 0}$ is ω-limit compact.

Additional to the results described in Lemma 2.3 and Lemma 2.4, we get the existence of the global attractor $\mathcal{A}$ to the problem (1.10) following the results given in [4]. □

3 CONCLUSIONS

We based on the energy estimates and built on the M.Conti's results by removing the technical conditions imposed on the memory kernels to deal the difficulties when considering the non-classical diffusion equation lacking instantaneous damping with hereditary memory. The result of the paper is proving the existence of bounded absorbing sets in spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, then prove the existence of global attractors $\mathcal{A}$.

REFERENCES

[1] E.C. Aifantis, On the problem of diffusion in solids, *Acta Mechanica* 37 (1980), 265-296.

[2] M. Conti, F. Dell'Oro and V. Pata, Nonclassical diffusion equation with memory lacking instantaneous damping, *Communications on Pure & Applied Analysis* 19 (2020) 2035-2050.

[3] C.M. Dafermos, Asymptotic stability in viscoelasticity, *Archive for Rational Mechanics and Analysis* 37 (1970), 297-308.

[4] Q.F. Ma, S.H. Wang and C.K. Zhong, Necessary and sufficient conditions for the existence of global attractor for semigroup and application, *Indiana University Mathematics Journal*, 51, (2002) 1541-1559.

[5] N.D. Toan, Existence and long-time behavior of variational solutions to a class of nonclassical diffusion equations in non-cylindrical domains, *Acta Mathematica Vietnamica* 41 (2016), 37-53.

[6] Y. Xie, Y. Li, Y. Zeng, Uniform attractors for nonclassical diffusion equations with memory,

Journal of Function Spaces (2016), Art. ID 5340489, 11 pp.

[7] Y. Xiao, Attractors for a nonclassical diffusion equation, *Acta Mathematicae Applicatae Sinica* 18 (2002), 273-276.

[8] F. Zhang and Y. Liu, Pullback attractors in $H^1(\mathbb{R}^N)$ for non-autonomous nonclassical diffusion equations, *Dynamical Systems* 29 (2014), 106-118.

[9] Y. Zhang, X. Wang, C. Gao, Strong global attractors for nonclassical diffusion equation with fading memory, *Advances in Difference Equations* 2017, Paper No. 163, 14 pp.