TẠP CHÍ KHOA HỌC ĐẠI HỌC TÂN TRÀO

ISSN: 2354 - 1431

http://tckh.daihoctantrao.edu.vn/

IMPROVING ALTERNATIVE THINKING IN DETERMINING THE INTERSECTION OF CURVED SURFACES BASED ON AUXILIARY SPHERE METHOD

Nguyen Thu Huong, Nguyen Thi Thu Nga*

Ha noi University of Science and Technology, Viet Nam

Email address: huong.nguyenthu@hust.edu.vn

https://doi.org/10.51453/2354-1431/2023/992

Article info

Received: 10/6/2023

Revised: 26/7/2023

Accepted: 08/10/2023

Keywords

Intersection; sphere; supporting surface; supporting sphere; CAD.

Abstract

Higher education aims to train high-quality human resources; the output products of higher education must have all the criteria in terms of knowledge, cognitive capacity, autonomy, and skills—ability, attitude, and responsibility. In order to achieve that output standard, students must have the ability for mathematical thinking to gradually improve their ability to design products in mechanical engineering. Geometry Drawing at engineering universities directs students to know the intersection of two surfaces and apply them to professional practice. Through teaching Geometry and Graphics, contributing to the development of algorithmic thinking for students, and helping them know how to use CAD software and apply mathematics to design and create technical details and space surfaces. This article demonstrates the method of finding the intersection between two curved surfaces using auxiliary spheres at different levels, from simple to complex, which consequently helps develop the students' geometric thinking as well as proficient use of the method of finding the intersection between two curved surfaces and utilization of algorithms in designing machine parts. Furthermore, this method is the foundation of building programming algorithms in AutoCad that can draw high-precision intersections and be applied in teaching and mechanical processing.

Vol 9. No 5_October 2023

TẠP CHÍ KHOA HỌC ĐẠI HỌC TÂN TRÀO

ISSN: 2354 - 1431

http://tckh.daihoctantrao.edu.vn/

NÂNG CAO TƯ DUY THUẬT TOÁN XÂY DỰNG GIAO HAI MẶT CONG BẰNG PHƯƠNG PHÁP MẶT CẦU PHÙ TRỢ

Nguyễn Thu Hương, Nguyễn Thị Thu Nga*

Đại học Bách Khoa Hà Nội, Việt Nam

Địa chỉ email: huong.nguyenthu@hust.edu.vn

https://doi.org/10.51453/2354-1431/2023/992

Thông tin bài viết

Ngày nhận bài: 10/6/2023

Ngày sửa bài: 26/7/2023

Ngày duyệt đăng: 08/10/2023

Từ khóa

Giao; mặt cầu; mặt phụ trợ; mặt cầu phụ trợ; CAD.

Tóm tắt

Mục tiêu của giáo dục đại học là đào tạo nguồn nhân lực chất lượng cao. Đầu ra của giáo dục đại học phải đáp ứng đủ các tiêu chí về kiến thức, năng lực nhận thức, tự chủ, kỹ năng, thái độ và trách nhiệm... Để đạt chuẩn đầu ra đòi hỏi người học phải có năng lực tư duy thuật toán, từng bước nâng cao khả năng thiết kế chế tạo sản phẩm trong thiết kế cơ khí. Môn Hình học họa hình ở các trường đại học khối kỹ thuật hướng người học tới việc biết xác định giao của hai mặt và bước đầu vận dụng vào việc thiết kế các chi tiết kỹ thuật. Bài báo trình bày phương pháp tìm giao của hai mặt cong bằng cách sử dụng mặt cầu phụ trợ theo các mức độ khác nhau từ đơn giản đến phức tạp, thông qua đó góp phần phát triển tư duy hình học cho người học, giúp người học có thể sử dụng thành thạo phương pháp tìm giao giữa các mặt cong và vận dụng thuật toán vào thiết kế chế tạo chi tiết máy. Đồng thời, phương pháp cầu phụ trợ cũng là cơ sở để xây dựng thuật toán lập trình trong AutoCad giúp vẽ giao có độ chính xác cao, ứng dụng trong giảng dạy và gia công cơ khí.

1. Introduction

In the real world, there are many objects and phenomena that people haven't fully perceived yet. The task of life always requires people to understand these problems to point out their essence and rules. According to psychology [1], thingking develops from intellectual operations: analysis- synthesis, comparison- similarity, abstraction- generalization. To gradually improve the mathematical thinking applied in the formation and construction of product design ideas for learners, the article presents a method of using auxiliary spheres to solve some common intersection problems in actual product design.

In the textbooks of Graphic Geometry [2], [3], [4], [5], the auxiliary plane method has been used to find the intersections of the objects such as finding the intersection of a plane with a curved surface, finding the intersection of two planes, or the intersection of a straight line with a curved surface. In addition, in [6], the author also introduced the method of a sphere inscribed in a cone to solve some problems of geometry construction, distance measurement.

There are alot of different methods to find the intersection of two curved surfaces such as using projection transformations, using auxiliary cutting surfaces or using analytical methods. Each method has its own advantages and disadvantages. For example, projection transformations such as changing the projection plane, replacing the projection, rotating the projection plane... are easy to understand but it takes learners more time to solve the problem and have to use more intermediate steps, and these methods can only solve simple cases of curved surface intersections. These methods can not help learners improve their thinking in geometry and visualize the shape of objects in the design. Analytical method can be used to find intersection of two curved surfaces with general equation F1(x, y, z) = 0 and F2(x, y, z) = 0 by parameterization. The parametric equations of the curve that is intersection of two curved surfaces will have the form $x = f(t)$, $y = g(t)$, $z = h(t)$. Obviously, using the analytical method to find intersections of curved surfaces does not provide learners with intuitive images, and it can not improve learners' geometric thinking in design.

The references show that finding intersection of two surfaces using auxiliary surfaces opens many opportunities to develop algorithmic thinking, specifically:

- Most of the intersection problems can be reduced to basic algorithms and problems, so learners can repeat them many times until they are proficient.

- The problems of finding intersections can be classified and arranged from simple to complex, from easy to difficult to facilitate the development of learners' algorithmic thinking.

- The problems of finding intersections need to be solved in the correct logical sequence. That is the basis for the formation of the algorithm. Thus, the algorithmization of the problem solving process will strengthen the learners' fundamental knowledge.

- The problems of finding intersections can be solved in many different ways. These problems will help learners to propose many algorithms to solve.

2. Research method using auxiliary spheres

This paper proposes a method using auxiliary spheres to find intersections between curved surfaces in order for learners to improve algorithmic thinking and flexibly use this method to find intersections. The problems are given from simple to complex. The content of this method is that the auxiliary spheres are used as an intermediate step to help solve the problems of finding intersection more easily.

This method is as follows [6], [7], [8], [9]: To determine the point M at the intersection of any two surfaces ω and θ, these surfaces are intersected by an auxiliary surface R (this auxiliary surface can be a plane or a curved surface). The auxiliary surface is chosen in such a way that it intersects both surfaces ω and θ, resulting in two auxiliary intersections with easily drawable projections using a compass and a ruler: $t = \omega \cap R$, $l = \theta \cap R$. Then t and l lie on the same surface R, they intersect at the desired point M. (Figure 1)

The paper presents how to find the intersection of two curved surfaces using the auxiliary sphere method from simple to complex. The auxiliary sphere used has a fixed and movable center that is suitable for learners to improve geometric and algorithmic thinking. The method using the auxiliary sphere is divided into iterative steps to systematize the method and create a suitable loop for programming by Autolips. The systematization of steps using the auxiliary sphere method helps learners master the method, use it fluently and improve their algorithmic thinking.

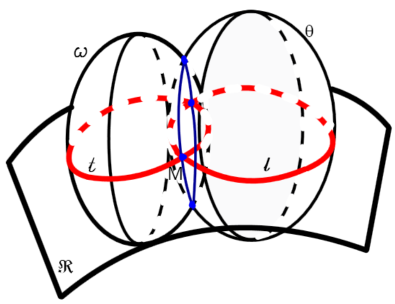

Figure 1: Finding the intersection of two curved surfaces by the auxiliary surface

3. Some problems of finding intersection of two curved surfaces using the method of auxiliary spheres

3.1. Concentric auxiliary spheres

Problem 1: Draw the intersection curve of a rotated circular surface with a sphere. The rotated circular surface has its axis perpendicular to the top view plane P_2

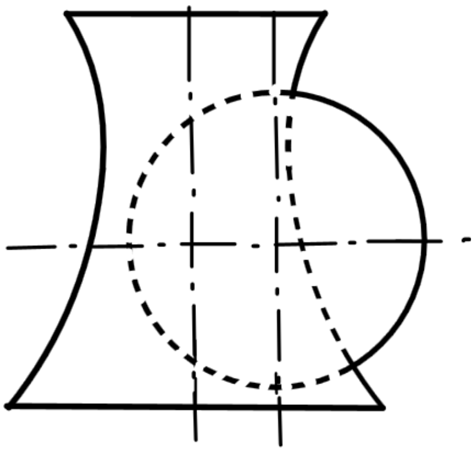

Figure 2a: Graphs of a rotated circular surface and sphere.

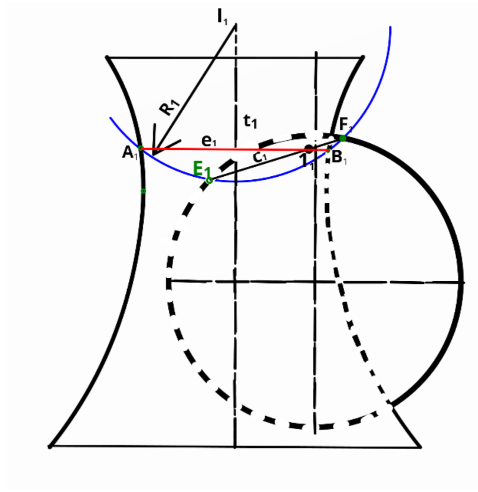

Figure 2b: Graph of how to find a point of the intersection

Solution:

Step 1: Divide the problem into simpler cases as shown in Figure 2b. Choose a fixed point I (the front view I_1 is on the axis of the rotated circular surface) as the center of auxiliary spheres whose radius is R_1. The radius R_1 satisfies the condition that the auxiliary sphere intersects both curved surfaces at the two auxiliary intersections which are circles. The auxiliary sphere intersects the rotated circular surface at a circle whose front view is e1 and intersects the given sphere at another circle whose front view is c_1. Both lines e_1 and c_1 intersect at 11 which is the sought-after point.

Step 2: The intersection of the curved surfaces is constructed by algorithmicization and generation of iterative steps. A series of concentric auxiliary spheres with center point I is constructed consecutively, the auxiliary sphere radius Ri changes (i=1,2,3…n) so that on the top view plane of this radius satisfies $R_{min} < R_{1i} < R_{max}$. By repeating the above method, the set of intersection points forms the desired curved intersection. (Figure 2c)

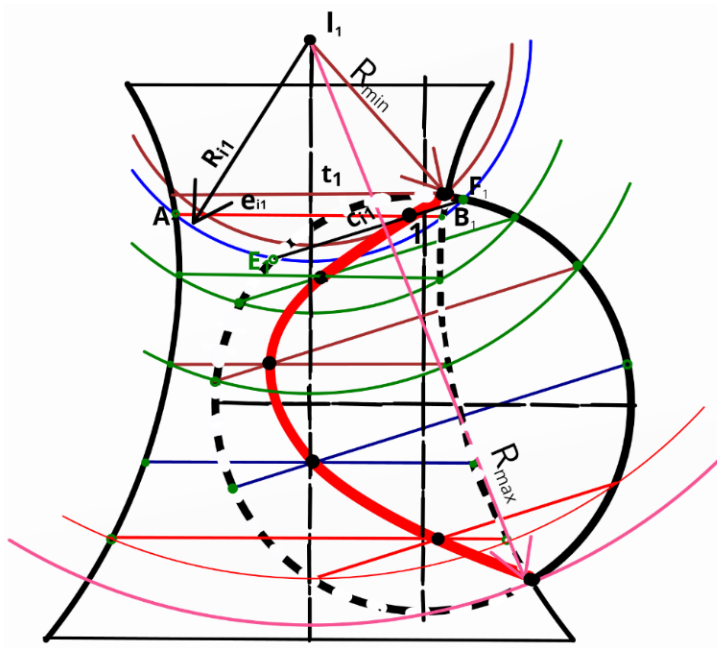

Figure 2c: Graph illustrating the method of finding intersection points and the intersection of two surfaces

Problem 2: Draw the front view of the intersection between a cone and a cylinder surface with their axes intersecting at O. The plane containing the two axes is parallel to the front view plane P_1 (Figure 3a).

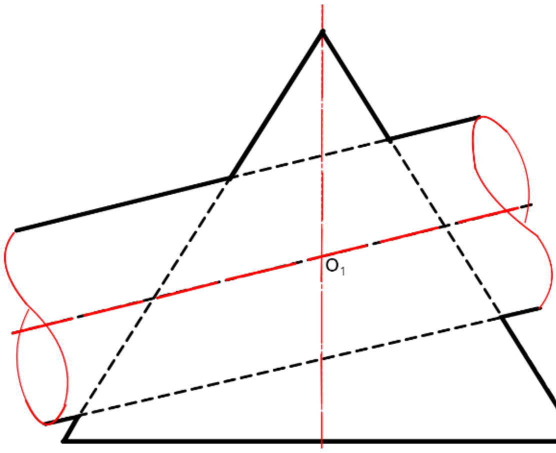

Figure 3a: Graph of a cone and a cylinder

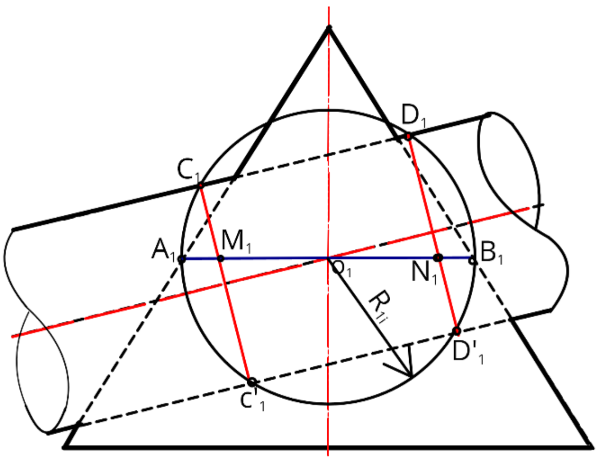

Figure 3b: Graph of how to find a point of the intersection

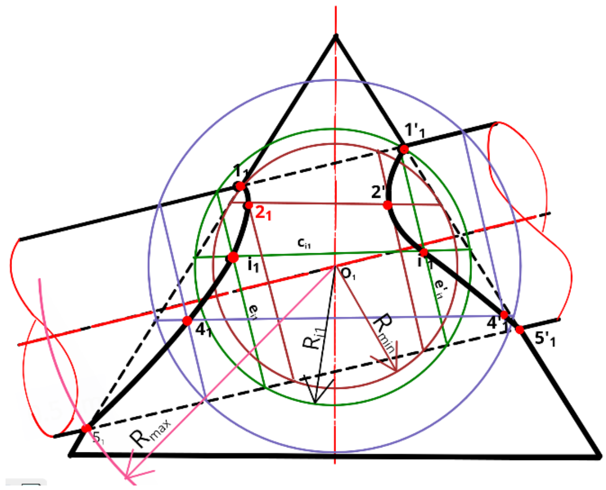

Figure 3c: Graph of how to find points of the intersection and the intersection of two surfaces

Solution:

Step 1: Divide the problem into smaller subproblems. The fundamental steps correspond to specific construction positions (Figure 3b). Take center O with front view O1 as the center of auxiliary sphere of radius R1i. This sphere intersects the cylinder at two circles whose front views are segments $C_1C'_1$ and $D_1D'_1$. At the same time, this sphere also intersects the cone at a circle whose front view is the segment A_1B_1 . On the front view, A_1B_1 intersects $C_1C'_1$ and $D_1D'_1$ at M and N which are the sought-after points.

Step 2: Step 1 is algorithmized (Figure 3c) to build a loop to find the points of the intersection. The center of the auxiliary sphere O does not change while the auxiliary sphere radius R_{1i} changes through iterative steps i =1, 2, 3, 4 … n satisfying the condition Rmin < R1i < Rmax.

3.2. Non-concentric Auxiliary Spheres

In order to improve learner' thinking and application of algorithms in finding more difficult intersections, the auxiliary sphere method can also be applied in some other cases, using non-concentric auxiliary spheres. The algorithm used in this case has a moving auxiliary sphere center and a variable auxiliary sphere radius as well.

Problem 3: Draw the intersection of a truncated rotating cone and a totus. They have skew axises and the graph is shown in the figure. (Figure 4a)

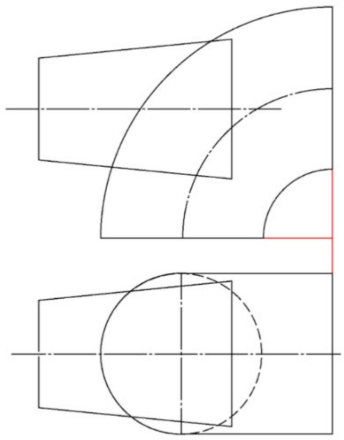

Figure 4a: Graph of a cone and a totus

As given, the l-axis of the cone is parallel to the front view plane P1, and the k-axis of the torus is perpendicular to P_1 (Figure 4b). k and l are intersected and perpendicular to each other. It is known that every plane α passing through the k-axis of the torus intersects the torus in a circle. Let's assume that plane αi intersects the torus along the M_iN_i generating circle which have the top view M_{i1} N_{i1}. From the center of the generating circle, draw a line perpendicular to the plane α, which intersects the axis l of the cone at point I with the top view I_1. Using I as the center, draw an auxiliary sphere passing through the circle MiNi. This sphere intersects the cone surface at E_iF_i with its top view $E_{i1}F_{i1}$. $E_{i1}F_{i1}$ and $M_{i1}N_{i1}$ intersect at i_1 which is on the intersection of two curved surfaces. The plane α_i is constructed step by step with i = 1, 2, 3, 4, ... so that $\alpha_{min} < \alpha_i < \alpha_{max}$.

The intersection of two curved surfaces is constructed by connecting the found intersection points sequentially.

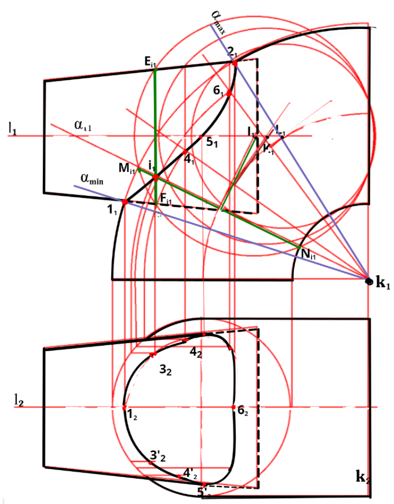

Figure 4b: Graph of how to find points of the intersection and the intersection of two surfaces

Problem 3 presents a high level of difficulty. The auxiliary spheres change both in center and radius. However, the problem can be solved with simple step decomposition and logical loop construction. Clearly defining the step and loop of the construction steps helps learners to find the intersection of two curved surfaces. On the basis of the loop of this method, programming by Autolips can be applied to draw the intersection of two curved surfaces with the help of a computer.

3.3. Applications of the Method in Product Design Thinking

In mechanical product design, accurately determining the positions and shapes of intersection curves between surfaces is quite important. Based on those intersection shapes, the bearing capacity and durability of machine parts during operation are determined. Geometric thinking through the method of determining the intersection correctly helps the designer to accurately assess their design, allowing adjustments in terms of shape or size to achieve an aesthetically pleasing and well functioning design. The auxiliary sphere method is built by repeating the construction steps, resizing the auxiliary face. Fundamentally, this is a principle used to construct loops in design and fabrication programming in CAD system [10]. Currently, there are many software that support the design and drawing of intersections, but they only support drawing and rendering, so we can not master the database. On the other hand, the design support software only draws the intersection between the curved surfaces by drawing approximate curves, so the anchor points on the intersection are not accurate. By using the method of auxiliary spheres, the authors are researching a module to draw intersections of curved surfaces using the Autolisp language within AutoCad. This module helps learners clearly understand how to find the intersection of two curved surfaces intuitively with high accuracy, consequently improving geometric thinking. The coordinates of the points belonging to the intersection and the contour can be determined easily by this method. It aids in development of curved surfaces and can be applied in teaching, mechanical processing such as automatically cutting shapes in plastic deformation processing, automatic welding thereby mastering the database in the design and processing of mechanical components.

4. Conclusion

The paper presents a method of using auxiliary spheres to draw the intersection between quadratic surfaces, which are common surfaces in mechanical design. This method aims to enhance geometric thinking and mathematical reasoning in the design of mechanical components. Precisely determining the specific shapes of intersections contributes to enhancing the technical accuracy in each product. This research provides an additional method to help learners develop mathematical thinking, specifically in geometric design of product surfaces. This method supports designers and engineers in enhancing their practical geometric thinking when using CAD software to finalize products with high accuracy, that is suitable for manufacturing capabilities, ensuring manufacturing quality and economic efficiency. Based on the method of finding the intersection of two curved surfaces, users can move towards mastering the database, and it can open up new research directions in the development of curved surfaces with complex intersections, programming the coordinates of intersection points, and applying them in mechanical processing and CNC cutting.

REFERENCES

[1] Hac, P.M. (1998), Textbook of Psychology Education Publishing House.

[2] Gary R.Bertoline, Eric N.Wiebe, (2000), Technical Graphics communication.

[3] Nhuan, P.V. (2005), How to solve some advanced problemsin Graphic Geometry, Bach khoa Publishing House.

[4] Hien, N.V. (2003), Descriptive Geometry, Science and Technology Publishing House

[5] Hyukhin S, (2007), Modeling the profile of the cylindrical surfaces that are machined with disc tools, Russ Eng Res 27

[6] Nhuan, P.V. (2005), Method of sphere inscrebed in cone, Bach khoa Publishing House.

[7] Ratko Obradovic, (2000), Determination of intersecting curve two surfaces of revolution with intersecting axes by use of auxiliary spheres. Facta universitatis- series Architecture and Civil Engineering vol.2, No2, pp 117-129.

[8] Ratko Obradovic, (1999), Surfaces-surface intersection: Auxiliary spheres, Novi Sad Journal of Mathematics, Vol 29, No3, pp 221-230.

[9] Ratko Obradovic, (2002), Determination of intersecting curve two surfaces of revolution with intersecting axes with parallel by use of auxiliary planes and auxiliary spheres, Facta universitatis- series Architecture and Civil Engineering, vol.2, No4, pp 267-272.

[10] Loc, N.H. (2001), AUTOLIPS Progamming Language, Hồ Chí Minh Publishing House.